

MTH 30 LECTURE NOTES (Ojakian)

Topic 21: Graphing the trigonometric functions

OUTLINE

(References: 6.1, 6.2)

1. General Points on graphing trig functions

- (a) The input: Angles in radians (all values allowed: 0, negative, positive).
- (b) Strategy: At least calculate every $\pi/2$ step.

2. Basic graphs of sin and cos

Questions: What do sound and light “look like?” How do we explain difference in pitch of sound? How do we explain color?

PROBLEM 1. Graph $y = \sin(x)$.

PROBLEM 2. Graph $y = \cos(x)$.

PROBLEM 3. Make some observations about the sin and cos graph.

- (a) What is the domain?
- (b) What are the possible values?
- (c) In what way are the graphs repetitive?

3. Period of a graph

(Roughly) to find the **period** of a graph (if it exists!):

- (a) Choose a piece of the graph.
- (b) Can you get the whole graph by shifting this piece to the left and right?
- (c) Is this the shortest piece that works?
- (d) Then, the length of this piece is the period of the graph.

4. Old graph Transformations on sin and cos

Apply old graph transformations to sin and cos

- (a) Vertical Shift
- (b) Horizontal Shift
- (c) Reflection: Across y axis
- (d) Reflection: Across x axis

5. New Graph Transformations

- (a) Vertical Stretch/Compression (multiplication outside function)
 - i. Larger than 1: Stretch
 - ii. Between 0 and 1: Compression
- (b) Horizontal Stretch/Compression (multiplication inside function)
 - i. Larger than 1: Compression
 - ii. Between 0 and 1: Stretch

6. General Form For sin and cos

General Form: $y = A \sin(B(x - k)) + C$

- (a) May need to rewrite $\sin(Bx - C) = \sin(B(x - \frac{C}{B}))$, i.e. so $k = \frac{C}{B}$
- (b) Amplitude = A
- (c) Period = $\frac{2\pi}{|B|}$
- (d) Midline: $y = C$
- (e) Phase shift = k
(positive/right if k is positive; negative/left if k is negative). Watch out! ...
- (f) Exercises Section 6.1: 15, 16, 17

7. Graphing Tan Function

Recall: $\tan(x) = \frac{\sin(x)}{\cos(x)}$

- (a) New issue: Asymptotes
- (b) Period Change
- (c) Graphing Strategy:
 - i. Mark the number line at $\pi/2$ steps
 - ii. Put sin on top and cos on bottom
 - iii. For each function, mark its zeros and when positive and negative
- (d) Problems: 6.2: 22 - 26

8. Applications of Waves

- (a) Sound waves
 - i. Frequency: cycles per second (this measurement is Hertz).
 - ii. See:
<http://www-users.math.umn.edu/~rogness/math1155/soundwaves/>
- (b) Light waves
 - i. The frequency of visible light is referred to as color, and ranges from 430 trillion hertz, seen as red, to 750 trillion hertz, seen as violet.
 - ii. See:
<https://science.howstuffworks.com/light4.htm>

9. Exercises

- (a) From graph to info/equation. Section 6.1: 23 - 30.
- (b) Application. Section 6.1: 48.