MTH 30 LECTURE NOTES (Ojakian)

Topic 20: Expanding the trigonometric functions

OUTLINE

(References: 5.4, 5.2, 5.3)

1. Angles in standard position
2. Evaluating trig functions for any angle

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1. First define on triangle
   (a) The 6 trig functions on angles between 0 and 90 degrees.
   (b) $\sin, \cos, \tan$ and reciprocals: $\csc, \sec, \cot$
2. Solving Triangles
   (a) Solving a right triangle when you know TWO sides:
      i. Just the Pythagorean theorem!
   (b) Solving a right triangle when you know ONE side and ONE angle:
      i. Apply a trig function of the angle which includes the *unknown* side and a *known* side.
      ii. Solve for the unknown side.
   (c) Like 5.4: 10 to 16

   PROBLEM 1. *Solve each right* $\triangle ABC$ *using the given information. In each case* $m\angle C = 90^o$.

   i. $m\angle A = 80^o, b = 72$.
   ii. $m\angle A = 30^o, c = 33$.
3. Applications
   (a) Devise a strategy for finding the distance across a lake?
   (b) Devise a strategy for finding the distance across a river? (what is the *relevant* difference between a lake and a river)
   (c) Like 5.4: 46 to 51, 52 to 56

   PROBLEM 2. *The angle of elevation of the top of a fir tree is* 68^o *from an observation point 70 ft. from the base of the tree. Find the height of the tree.*

   PROBLEM 3. *The angle of depression from the top of the Empire State Building to a hot dog stand is* 60^o. *How far away is the hot dog stand? (what non-mathematical missing piece of information do we need?)*

4. Why expand trig functions?

Consider graphing sine between 0 and 90. What happens if we continue?

(a) Sound waves

(b) Light waves

(c) Length of a day over a year

(d) Etc!

5. Representing angles with any measure

(a) Standard position:

i. One side (the **initial side**) points in the positive x-direction.

ii. The other side (the **terminal side**) rotates from the initial side.

iii. Positive angle: counter-clockwise rotation.

iv. Negative angle: clockwise rotation.

(b)

PROBLEM 4. *Draw the following angles in standard position:*

$$45^o, -45^o, 270^o, -90^o, 135^o, 120^o, 450^o, \pi$$

(c)

Definition 1. *Angles with the same terminal side are called* ***co-terminal****.*

PROBLEM 5. *Which of the above angles are co-terminal. Find yet another angle co-terminal with them.*

6. Evaluating trig function from a point on terminal side

$\sin(\theta) = y/r$

$\cos(\theta) = x/r$

$\tan(\theta) = y/x$

$\csc, \sec$,and cot are defined as the reciprocals.

PROBLEM 6. *Evaluate the following:*

(a) $\tan(\theta)$ *and* $\sin(\theta)$ *where* $(-3, -4)$ *is a point on the terminal side of* θ*.*

(b) $\cot(\theta)$ *and* $\sec(\theta)$ *where* $(-1, 4)$ *is a point on the terminal side of* θ*.*

PROBLEM 7. *Evaluate the following:*

(a) $\sin(90^o)$

(b) $\cos(-180^o)$

(c) $\tan(-\pi/2)$

7. When the terminal point is harder to find

   (a) Use pythagorean theorem to find.

      i. For multiple of 45 degrees: $x = y$

      ii. For multiple of 30 or 60: One of x or y is 1/2. Find other.

   (b)

   PROBLEM 8. *Evaluate the following (use a special triangle if needed):*

   i. $\tan(225^o)$ *and* $\sin(225^o)$

   ii. $\cos(-210^o)$ *and* $\sec(-210^o)$

8. Evaluating trig function using a reference angle

   An alternative approach to evaluating trig functions of angles whose terminal side is not on the x or y axis.

   (a) Draw the angle in standard position.

   (b) Find the Reference Angle (the angle between the terminal side and the x-axis).

   (c) Evaluate the trig function at the Reference Angle.

   (d) Leave the answer as is, or modify by making negative, based on:
   The quadrant of the terminal side and the trig function being evaluated.

   (e)

   PROBLEM 9. *Evaluate the following:*

   i. $\cos(135^o)$, $\sin(135^o)$, *and* $\tan(135^o)$

   ii. $\sin(7\pi/6)$ *and* $\csc(7\pi/6)$

9. Evaluating trig expressions

   PROBLEM 10. *Evaluate the following.*

   (a) $\tan\frac{\pi}{4} + \cot\frac{7\pi}{4}$

   (b) $\sec^2\frac{5\pi}{6}$