

MTH 28 LECTURE NOTES (Ojakian)

Topic 16: Radical Arithmetic

OUTLINE

References: 8.4, 8.5

1. Rules

(a) Radicals

- i. $\sqrt[n]{A \cdot B} = \sqrt[n]{A} \cdot \sqrt[n]{B}$
- ii. $\sqrt[n]{\frac{A}{B}} = \frac{\sqrt[n]{A}}{\sqrt[n]{B}}$
- iii. $\sqrt[n]{A^n} = A$ (for $A \geq 0$)

(b) Exponents

- i. $A^n \cdot A^m = A^{n+m}$
- ii. $\frac{A^n}{A^m} = A^{n-m}$
- iii. $(A \cdot B)^n = A^n B^n$
- iv. $\left(\frac{A}{B}\right)^n = \frac{A^n}{B^n}$
- v. For $A \geq 0$: $(A^n)^m = A^{nm}$

2. Simplification Requirements

(a) *No exponents are negative.*

Example: $x^3 y^{-4}$ should be simplified to $\frac{x^3}{y^4}$

(b) *Nothing can be pulled out of a radical.*

Example: $\sqrt[3]{16}$ should be simplified to $2\sqrt[3]{2}$.

(c) *All the exponent rules (from the key rules above) should be carried out.*

Example: $x^3 x^5$ simplifies to x^8 .

(d) *Exponents on numbers should be evaluated, left as a simplified radical if need be.*

Example: $25^{1/2}$ should be simplified to 5.

Example: $8^{1/2}$ should be simplified to $2\sqrt{2}$.

(e) *There are no radical symbols in the denominator.*

Example: $\frac{1}{\sqrt{5}}$ should be simplified to $\frac{\sqrt{5}}{5}$

3. Division - Radical Expressions

PROBLEM 1. *Simplify the following:*

(a) $\frac{3}{\sqrt{6}}$

(b) $\sqrt{\frac{18m^5n^6}{p^{12}q^4}}$

(c) $\sqrt{\frac{4}{49AC^2}}$

(d) $\frac{10mn}{\sqrt{5mn}}$

4. Add/Subtract - Radical Expressions

PROBLEM 2. *Simplify the following.*

(a) $\sqrt{2} + 3\sqrt{2}$

(b) $\sqrt[3]{4} - 5\sqrt[3]{4}$

(c) $\sqrt{75} + \sqrt{48} - 7\sqrt{12}$

(d) $\sqrt{20} + \sqrt[3]{54} - 2\sqrt{45} + \sqrt{7} + 10\sqrt[3]{16} - 4\sqrt[3]{3000}$

(e) $3\sqrt{8}(2\sqrt{10} - \sqrt{27})$

(f) $(\sqrt{5})^2 + (\sqrt{3})^2$

(g) $(\sqrt{5} + \sqrt{3})^2$