

MTH 23.5 LECTURE NOTES (Ojakian)

Topic 18: Hypothesis Testing

OUTLINE

References (**Algebra Book**: None; **Statistics Book**: ch 9)

1. Hypothesis Testing
 2. P-Values
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**Note: We will not refer to the “sample test statistic”
We are only covering the case where the standard deviation is known**

1. Confidence Intervals versus Hypothesis Testing

- (a) Confidence Interval: Estimate the value of the mean (with some confidence)
- (b) Hypothesis Testing: Make decision about the value of the mean (with some confidence)

2. Hypothesis Testing: The Setup

- (a) First Example:
See Example 1 from Section 9.1 (5th edition, p. 353)
 - i. Null Hypothesis (H_0): The hypothesis to test
 - ii. Alternative Hypothesis (H_1)
- (b) Second Example:
OpenStax book page 507: Example 9.2 and Exercise 9.2
- (c) Third Example:
Exercise 7 (a, b, c, d) from section 9.1 (5th edition, p. 364)
- (d) General points
 - i. Null Hypothesis is an “Equality Statement”
 - ii. Alternative Hypothesis can chosen to be any of following:
 - A. Not equal (“two-tailed”)
 - B. Less than (“left-tailed”)
 - C. Greater than (“right-tailed”)

3. P-Values

Probability the sample mean is as extreme as it is under the assumption of the null hypothesis.

Example: See example 2 (Section 9.1 of 5th edition, p.355)

- (a) Starting point: We have a normal distribution x with an **unknown** mean μ and a **known** standard deviation σ (or may be approximately normal).
- (b) The Assumption: Assume that the null hypothesis ($H_0 : \mu = 115$) is true for all calculations. What are the implications of this assumption???
- (c) Consider the sampling distribution \bar{x} .
 - i. Under our assumption, its mean is 115
 - ii. Under our assumption calculate the standard deviation as usual: σ/\sqrt{n}
- (d) $H_1 : \mu < 115$: P-Value = Probability of being less than $\bar{x} = \text{norm.dist}(\bar{x}, \mu, \sigma/\sqrt{n}, \text{true})$
- (e) $H_1 : \mu > 115$: P-Value = Probability of being more than $\bar{x} = \dots$ Do “1 minus ...”
- (f) $H_1 : \mu \neq 115$: P-Value = Probability of being less than \bar{x} or more than $\bar{x} =$ “TWO times left-tail case OR right-tail case” (which one, depends on the question!)
- (g)

PROBLEM 1. *From section 9.1 (5th Edition): Do exercise 5.*

Then compute the 3 kinds of P-Values for the assumption of a standard deviation of 5kg, and various samples.

4. Hypothesis Testing - Full Problem

- (a) Choose H_0 and H_1 (the null hypothesis and the alternative hypothesis)
- (b) Choose α (the *Level of Significance*).
Typical values: 0.05 and 0.01.
- (c) Calculate the P-Value: p
- (d) If $p > \alpha$ then accept H_0 . If $p < \alpha$ then reject H_0 (and accept H_1)
- (e) Error types:
 - i. Type 1: Rejecting H_0 when it is true.
 - ii. Type 2: Rejecting H_1 when it is true.
- (f)

PROBLEM 2. *From section 9.1 (5th Edition): Do exercise 10.*

PROBLEM 3. *From section 9.1 (5th Edition): Do exercise 11.*

PROBLEM 4. *From section 9.1 (5th Edition): Do exercise 13.*