

MTH 23.5 LECTURE NOTES (Ojakian)

Topic 17: Confidence Intervals

OUTLINE

References (**Algebra Book**: None; **Statistics Book**: 8.1)

1. Inverse Normal
 2. Finding confidence intervals using Excel
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1. Inverse Normal

PROBLEM 1. Consider a normal distribution with a mean of 15 and standard deviation of 3.

- (a) Find the x value, such that 0.5 of the area is to left of this x .
- (b) Find the x value such that 2.5% of the data is to the left (use empirical rule).
- (c) Find the x value such that 2.5% of the data is to the right (use empirical rule).

Use Excel: `Norm.inv(probability, mean, standard deviation)`

Do Webwork

2. Overview

- (a) Have some population and an unknown population mean μ .
- (b) Choose a “confidence level”: A percent, $P\%$, between 0% and 100%. Typical examples of choices: 90%, 95%, 99%.
- (c) Sample from the distribution in order to get an approximation (called \bar{x}) for μ .
- (d) Use that information to obtain a “ $P\%$ confidence interval (a, b) for μ ”.
- (e) The probability you pick an interval (a, b) that contains μ is $P\%$.
- (f) Example: Suppose the newspaper tells you that $(22.1, 25.8)$ is a 90% confidence interval for the average age of a college student. This means that you can be 90% confident that the average age of a college student is in between 22.1 and 25.8.

3. Confidence Intervals

Note how we are NOT doing it the way the book does it. We are using a simplified approach!

Note: we will always assume the standard deviation is known!

(a) Given Information:

- i. Given: You are given the standard deviation σ .
- ii. Given: You are given a confidence level c (as a percent or a decimal)
- iii. Given: You are given (or find) the sample mean \bar{x} (possibly a different letter is used)
- iv. Given: You are given the n corresponding to the sample mean.

(b) Calculations:

- i. Find the standard deviation of the sampling distribution $= \frac{\sigma}{\sqrt{n}}$
- ii. Consider the normal distribution with mean \bar{x} and the standard deviation from the last line.
- iii. For this distribution, find the interval (*LEFT*, *RIGHT*) centered at \bar{x} which contains c of the area:
 - A. Find the excluded area on each side $= \frac{1 - c}{2}$
 - B. *LEFT* = norm.inv(excluded area, mean, SD)
 - C. *RIGHT* = norm.inv($c +$ excluded area, mean, SD)

(c) Other points:

- i. **error tolerance:** E = distance from \bar{x} to either the left or right endpoint.
- ii. If data known to be normally distributed, then this works for any size n ; otherwise, a rule of thumb: n should be at least 30.

(d)

PROBLEM 2. *Suppose that for some data $\bar{x} = 2000$ and $\sigma = 100$. Suppose we are computing a confidence interval with confidence level 0.90. Find three confidence intervals, one for each of the following values of n : 30, 80, 500. Also find the error tolerances.*

What can you say in general?

PROBLEM 3.

- i. Suppose you are computing a 92% confidence interval. How much total area is excluded from consideration in your normal distribution? How much is excluded on the left? How much on the right?*
- ii. For a normal distribution, suppose the error tolerance is 5 and the sample mean is 20. What is the corresponding confidence interval?*
- iii. For a normal distribution with mean 10 and standard deviation 2, find an interval around the mean which includes 80% of the area of the curve.*

PROBLEM 4. *From section 8.1 (5th edition): Exercise 15.*