MTH 23.5 LECTURE NOTES (Ojakian)

Topic 10: More Probability

OUTLINE

References (**Algebra Book**: None; **Statistics Book**: 3.2, 3.3)

1. And, Or operations
2. Mutually exclusive
3. Conditional Probability
4. Independant

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1. An Example

(a) These days, what are the biggest dangers for people living in the USA?

(b) See Death statistics.

i. Think of sample space as: Different ways to die.

ii. Take frequency analysis point of view: Use previous data to compute probabilities.

2. Operations on Events

(a) Already saw complement.

(b) "*And*" Operation (Intersection of events)

PROBLEM 1. *Consider a 6-sided die which you roll once.*

i. Let A be the event: The roll is even.
Let B be the event: The roll is less than 5.
Find $P(A \text{ and } B)$.

ii. Find P(Even and Odd)

(c) "*Or*" Operation (Union of events)

PROBLEM 2. *Do the last problem, but replace the word "and" by the word "or".*

PROBLEM 3. *Consider the following attempt at a rule:*

$$P(A \text{ or } B) = P(A) + P(B)?!?!$$

Is the rule true or false? Use some examples to get evidence of your view.

PROBLEM 4. *Create a proper rule for $P(A \text{ or } B)$.*

PROBLEM 5. *You roll two fair 4-sided dice, one green and one blue.*

i. What is the probability of getting a sum of 3?

ii. What is the probability of getting a sum of 6?

iii. What is the probability of getting a sum of 3 or 6?

iv. What is the probability of getting a sum of 6 or having at least one of the die being a 4.

(d) Define: Mutually Exclusive (or disjoint)

PROBLEM 6. *Simplify the above rule for mutually exclusive events.*

3. Conditional Probability

$P(A|B)$: Probability of event A happening given that event B happens.

(a) Informal Idea (via examples)

i. P(Dying of esophageal cancer | Live in USA) = 0.00004
P(Dying of esophageal cancer | Live in Central Asia) = 0.00140

ii. P(Dying of lung cancer | Male and smoker) is 22 times larger than:
P(Dying of lung cancer).

(b) Mathematical Examples.

To find $P(A|B)$, limit yourself to the outcomes in B and find the percent of those that are also in A.

PROBLEM 7. *Suppose you flip a fair coin three times.*

i. What is the probability of 3 heads?

ii. What is the probability of 3 heads given that at least two flips are heads?

iii. Calculate P(Exactly two heads | At least one tail).

(c) Mathematical Formula.

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

PROBLEM 8. *Do ones from the last problem, using the formula.*

4. Independence

(a) Intuition: Two events are independent if the occurrence or nonoccurrence of one event does not change the probability that the other event will occur.

i. Intuitive Examples: Independent or not?

A. It rains in Manhattan versus It rains in The Bronx.

B. It rains in Manhattan versus It rains in San Juan.

ii. Mathematical Examples. Independent or not?

A. Roll a blue die and a red die. The blue die being 5 versus The red die being 2.

B. Roll one die: Rolling a two versus Rolling an even number.

C. Roll one die: Rolling an odd number versus Rolling an even number.

(b) Mathematical Definition.

Two events A and B are independent if $P(A \text{ and } B) = P(A) \cdot P(B)$

PROBLEM 9. *Roll one die one time. Let A be: The value is 2. Let B be: The value is an even number. Are A and B independent or not?*

PROBLEM 10. Use the fact that the roll of one die is independent of the roll of another die. *Suppose two dice are rolled, one blue and one red; what is the probability that the blue die is a 3 and the red die is a 5?*

PROBLEM 11. *Suppose two dice are rolled; what is the probability that one die is a 3 and the other die is a 5?*

PROBLEM 12. *Suppose a die is rolled three times; what is the probability that the first roll is a 3, the second roll is a 1, and the final roll is a 1?*

(c) Why the definition makes sense.

If B has no effect on A, then $P(A|B) = P(A)$. And then what ... ?