Kerry Ojakian's MTH 23.5 Class
Due Date: Tuesday December 3

HW #3

General Instructions:

- Do **not** write answers and work on a printout of this homework. Put all answers and work on separate paper (no need to copy the question).
- Homework must be relatively neat.
- Homework exercises must be done in order (if you skip an exercise, still write down the number and leave some blank space).
- You must show all work.
- While you may work with other students or tutors, do not copy someone else's work or data, or have someone else do the work for you.

The Assignment

1. Let x be a random variable that represents the amount of sugar in the blood, detected by a doctor's test. Suppose that x is normaly distributed with mean 85 and standard deviation 25.

   Note: Use Excel only on part (b).

   (a) What is the probability that on a single test, the amount of sugar is less than 60? (do NOT use the Excel command)

   (b) *(Use Excel)* Suppose that the doctor takes 10 tests and finds the average: $\overline{x}$.

   i. Why is the distribution $\overline{x}$ normal?

   ii. What is the mean and standard deviation of $\overline{x}$?

   iii. What is the probability that $\overline{x}$ is less than 60?

   iv. Why is the last probability smaller than the probability from part (a), which also asks about 60?

2. Suppose we have the following data set:

$$X: \ x_1 = 2, \quad x_2 = -3, \quad x_3 = 2, \quad x_4 = 0, \quad x_5 = 4, \quad x_6 = -1$$

Evaluate the following:

(a) $\sum x$

(b) $\left(\sum x\right) / 6$

(c) What is the mean of the data?

(d) What is the median of the data?

(e) What is the mode of the data?

(f) What is the range of the data?

(g) What is the sample standard deviation of the data?

3. (a) Compute a 75% Chebyshev interval for some data that has mean 40 and standard deviation 5.

(b) For the Chebyshev interval you just computed, if you were told that it was a normal distribution, then how much data would be in the interval?

4. Suppose three 6-sided dice are rolled (each die is numbered 1 to 6). Use the fact the three rolls are independant!

(a) What is the probability that the three rolls in order are: 3, 1, 1.

(b) What is the probability that among the three dice you only get 5s and 6s?

5. Given P(A) = 0.3 and P(B) = 0.4 and P(A and B) = 0.2, calculate the following:

(a) $P(A|B)$

(b) $P(B|A)$

(c) $P(A \text{ or } B)$

6. In the USA, four year colleges have a graduation rate of about 60%, and two year colleges (like Bronx Community!) have a graduation rate of about 30% (yes, I believe these percentages are correct!).

   As a social note: these numbers are not great! (I hope you can do all you can, and we can help you do all you can, to be in that 30%!).

   (a) Suppose there are 16 two year college students in a class. What is the probability that 2 or fewer graduate? (show your work, showing the use of the formula)

   (b) Suppose there are 16 two year college students in a class. What is the probability that all but one graduate? (show your work, showing the use of the formula)

   (c) Suppose there are 10 students in a class at a four year college. What is the probability that they all graduate? (show your work, showing the use of the formula)

   (d) What are you going to do to make sure you're in the 30% ?!

7. Suppose that a store makes over $800 on 60% of the days.

   (a) Suppose the store is open for 20 days. What is the probability that the store will make over $800 every day?

   (b) Suppose the store is open for 10 days. What is the probability that the store will make over $800 for at least 3 days?

   (c) Suppose the store is open for 10 days. What is the probability that the store will make **less than** $800 for at least 3 days?

8. Suppose the police have a response time that is normally distributed with a mean of 8.5 minutes and a standard deviation of 1.5 minutes.

   Answer the following questions **using the Empircal Rule, without Excel**.

   (a) What is the probability that the response time is between 7 and 10 minutes?

   (b) What is the probability that the response time is less than 10 minutes?

   (c) What is the probability that the response time is less than 7 mintues?

   (d) What is the probability that the response time is **more** than 7 minutes?

9. Suppose X is normaly distributed with a mean of 30 and standard deviation of 5. Find the following using Excel (write down exactly what your Excel work is, and give the final answer).

   (a) $P(X < 28)$

   (b) $P(X > 26.5)$

   (c) $P(29 < X < 30.5)$

10. Suppose that the distance of fly balls hit to the outfield (in baseball) is normally distributed with a mean of 250 feet and a standard deviation of 50 feet; call this distribution X We randomly sample 49 fly balls.

    (a) If $\bar{X}$ = average distance in feet for 49 fly balls, then what is the mean and standardare deviation for $\bar{X}$?

    (b) What is the probability that the 49 balls traveled an average of less than 240 feet? Sketch the graph of the distribution $\bar{X}$ and shade the region corresponding to the probability.

    (c) Find the 80th percentile of the distribution of the average of 49 fly balls.

11. Suppose X is normaly distributed with a mean of 20 and standard deviation of 10. Suppose we take 4 samples, to form the sampling distribution $\bar{X}$. Use the empirical rule (without Excel!) to determine the following.

    (a) $P(10 < \bar{X} < 30)$

    (b) $P(\bar{X} < 10)$

    (c) $P(\bar{X} > 10)$

12. *(Use Excel)* Suppose a sample of some lions in California is taken and their weights (in pounds) are: 68, 104, 128, 122, 60, 64. We can calculate that $\overline{x} = 91$ and the standard deviation is 30.7 (you do not need to calculate these values; just trust that they are correct).

    Find a 80% confidence interval for the mean weight of lions in California.

13. Suppose μ represents the average height of men in The Bronx. Someone takes a random sample of 35 men from The Bronx and finds that the mean height of these 35 men is 68 inches, and the standard deviation for these 35 men is 5 inches.

    (a) Find a 70% confidence interval for μ.

    (b) **Without** a calculation answer this question: Suppose you stuck with the sample of 35 men, but wanted a 95% confidence interval. Would the 95% confidence interval be larger or smaller than the interval from part (a)? Justify your answer (briefly, without a calculation).