

# MTH 23 LECTURE NOTES (Ojakian)

## Topic 12: Hypothesis Testing

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### OUTLINE

References (**Algebra Book**: None; **Statistics Book**: ch 9)

1. Hypothesis Testing
  2. P-Values
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**Note: We will not refer to the “sample test statistic”**  
**We are only covering the case where the standard deviation is known**

#### 1. Confidence Intervals versus Hypothesis Testing

- (a) Confidence Interval: Estimate the value of the mean (with some confidence)
- (b) Hypothesis Testing: Make decision about the value of the mean (with some confidence)

#### 2. Hypothesis Testing: The Setup

- (a) First Example:  
See Example 1 from Section 9.1 (5th edition, p. 353)
  - i. Null Hypothesis ( $H_0$ ): The hypothesis to test
  - ii. Alternative Hypothesis ( $H_1$ )
- (b) Second Example:  
OpenStax book page 507: Example 9.2 and Exercise 9.2
- (c) Third Example:  
Exercise 7 (a, b, c, d) from section 9.1 (5th edition, p. 364)
- (d) General points
  - i. Null Hypothesis is an “Equality Statement”
  - ii. Alternative Hypothesis can choose to be any of following:
    - A. Not equal (“two-tailed”)
    - B. Less than (“left-tailed”)
    - C. Greater than (“right-tailed”)

#### 3. P-Values

Probability the sample mean is as extreme as it is under the assumption of the null hypothesis.

Example: See example 2 (Section 9.1 of 5th edition, p.355)

- (a) Starting point: We have a normal distribution  $x$  with an **unknown** mean  $\mu$  and a **known** standard deviation  $\sigma$  (or may be approximately normal).
- (b) The Assumption: Assume that the null hypothesis ( $H_0 : \mu = 115$ ) is true for all calculations. What are the implications of this assumption???

- (c) Consider the sampling distribution  $\bar{x}$ .
  - i. Under our assumption, its mean is 115
  - ii. Under our assumption calculate the standard deviation as usual:  $\sigma/\sqrt{n}$
- (d)  $H_1 : \mu < 115$ : P-Value = Probability of being less than  $\bar{x} = \text{norm.dist}(\bar{x}, \mu, \sigma/\sqrt{n}, \text{true})$
- (e)  $H_1 : \mu > 115$ : P-Value = Probability of being more than  $\bar{x} = \dots$  Do “1 minus ...”
- (f)  $H_1 : \mu \neq 115$ : P-Value = Probability of being less than  $\bar{x}$  or more than  $\bar{x} =$   
 “TWO times left-tail case OR right-tail case” (which one, depends on the question!)
- (g)

**PROBLEM 1.** *From section 9.1 (5th Edition): Do exercise 5.*

*Then compute the 3 kinds of P-Values for the assumption of a standard deviation of 5kg, and various samples.*

#### 4. Hypothesis Testing - Full Problem

- (a) Choose  $H_0$  and  $H_1$  (the null hypothesis and the alternative hypothesis)
- (b) Choose  $\alpha$  (the *Level of Significance*).  
 Typical values: 0.05 and 0.01.
- (c) Calculate the P-Value:  $p$
- (d) If  $p > \alpha$  then accept  $H_0$ . If  $p < \alpha$  then reject  $H_0$  (and accept  $H_1$ )
- (e) Error types:
  - i. Type 1: Rejecting  $H_0$  when it is true.
  - ii. Type 2: Rejecting  $H_1$  when it is true.
- (f)

**PROBLEM 2.** *From section 9.1 (5th Edition): Do exercise 10.*

**PROBLEM 3.** *From section 9.1 (5th Edition): Do exercise 11.*

**PROBLEM 4.** *From section 9.1 (5th Edition): Do exercise 13.*