MTH 23.5 LECTURE NOTES (Ojakian)

Topic 11: Confidence Intervals

OUTLINE

References (**Algebra Book**: None; **Statistics Book**: 8.1)

1. Inverse Normal
2. Finding confidence intervals using Excel

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1. Inverse Normal

PROBLEM 1. *Consider a normal distribution with a mean of 15 and standard deviation of 3.*

(a) Find the x value, such that 0.5 of the area is to left of this x.

(b) Find the x value such that 2.5% of the data is to the left (use empircal rule).

(c) Find the x value such that 2.5% of the data is to the right (use empiral rule).

Use Excel: `Norm.inv`(probability, mean, standard deviation)

Do Webwork

2. Overview

(a) Have some population and an unknown population mean μ.

(b) Choose a "confidence level": A percent, $P\%$, between 0% and 100%. Typical examples of choices: $90\%, 95\%, 99\%$.

(c) Sample from the distribution in order to get an approximation (called $\bar{x}$) for μ.

(d) Use that information to obtain a "$P\%$ confidence interval (a, b) for μ".

(e) The probability you pick an interval (a, b) that contains μ is $P\%$.

(f) Example: Suppose the newspaper tells you that $(22.1, 25.8)$ is a 90% confidence interval for the average age of a college student. This means that you can be 90% confident that the average age of a college student is in between 22.1 and 25.8.

3. Confidence Intervals

Note how we are NOT doing it the way the book does it. We are using a simplifed approach!

Note: we will always assume the standard deviation is known!

(a) Given Information:

i. Given: You are given the standard deviation σ.

ii. Given: You are given a confidence level c (as a percent or a decimal)

iii. Given: You are given (or find) the sample mean $\bar{x}$ (possibly a different letter is used)

iv. Given: You are given the n corresponding to the sample mean.

(b) Calculations:

i. Find the standard deviation of the sampling distribution $= \dfrac{\sigma}{\sqrt{n}}$

ii. Consider the normal distribution with mean $\bar{x}$ and the standardard deviation from the last line.

iii. For this distribution, find the interval $(LEFT, RIGHT)$ centered at $\bar{x}$ which contains c of the area:

A. Find the excluded area on each side $= \dfrac{1-c}{2}$

B. LEFT = norm.inv(excluded area, mean, SD)

C. RIGHT = norm.inv(c + excluded area, mean, SD)

(c) Other points:

i. **error tolerence**: E = distance from $\bar{x}$ to either the left or right endpoint.

ii. If data known to be normally distributed, then this works for any size n; otherwise, a rule of thumb: n should be at least 30.

(d)

PROBLEM 2. *Suppose that for some data* $\overline{x} = 2000$ *and* $\sigma = 100$. *Suppose we are computing a confidence interval with confidence level* 0.90. *Find three confidence intervals, one for each of the following values of* n*: 30, 80, 500. Also find the error tolerences.*

What can you say in general?

PROBLEM 3.

i. Suppose you are computing a 92% *confidence interval. How much total area is excluded from consideration in your normal distribution? How much is exluded on the left? How much on the right?*

ii. For a normal distribution, suppose the error tolerence is 5 and the sample mean is 20. What is the corresponding confidence interval?

iii. For a normal distribution with mean 10 and standard deviation 2, find an interval around the mean which includes 80% *of the area of the curve.*

PROBLEM 4. *From section 8.1 (5th edition): Exercise 15.*