MTH 23.5 LECTURE NOTES (Ojakian)

Topic 8: Binomial Distribution and Background

OUTLINE

References (**Algebra Book**: None; **Statistics Book**: 3.5, 4.3)

1. Tree Diagrams (3.5)
2. Binomial Distributions (4.3)

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1. Inequality Background

   Do problems like the first four webwork problems and ones like problem 5.

2. Calculation Background

   (a) Exponents

   (b) Factorial

   (c) $C_{n,r} = \dfrac{n!}{r! \cdot (n-r)!}$ = number of subsets of S of size r (S is any set with n elements).

   (d)

   PROBLEM 1. *Calculate the following:*

   i. $(0.4)^3$
   ii. $3!$
   iii. $1!$
   iv. $0!$
   v. $5 \cdot (10-8)!$
   vi. $C_{3,2}$
   vii. $(0.1)^4$
   viii. $4!$
   ix. $C_{4,1}$

3. Repeated Trials and Tree Diagrams

   (a) Note: WITH and WITHOUT replacement. Reprsent the following using tree diagrams.

   PROBLEM 2. *Two cards are drawn from a regular deck of 52 cards,* **with replacement**. *What is the probability that the first card is an ace and the second is a king? What is the probability the first card is NOT an ace and the second is a king?*

   PROBLEM 3. *Two cards are drawn from a regular deck of 52 cards,* **without replacement**. *What is the probability that the first card is an ace and the second is a king? What is the probability the first card is NOT an ace and the second is a king?*

   PROBLEM 4. *A jar contains 2 red marbles, 5 blue marbles, and 5 yellow marbles.*

i. *Suppose you take 2 marbles, without replacement. What is the probability that you take 1 blue and 1 yellow, in any order.*

ii. *Suppose you take 2 marbles, with replacement. What is the probability that you take 1 blue and 1 yellow, in any order.*

iii. *Suppose you take 3 marbles, without replacement. What is the probability that all are red?*

iv. *Suppose you take 3 marbles, without replacement. What is the probability that at least one is not red?*

4. Binomial Distribution

(a) Binomial Experiment Example

i. Flip the same coin 3 times and count the number of heads (biased so probability of heads is 0.2). What is the probability of all heads? What is the probability of 2 heads? Etc? Draw a probability tree diagram. Draw a histogram of the distribution.

ii. Parameters/Terminology
n = number of trails
p = probability of "success"
q = probability of "failure"
r = number of successes
$P(r)$ = probability of r successes
$P(r < x)$ = probabillity of less than x successes (ETC.)

iii. Using Excel

A. BINOM.DIST(r, n, p,FALSE): For probability of **exactly r successes**.

B. BINOM.DIST(r, n, p,TRUE): For probability of **r or fewer successes**.

PROBLEM 5. *Use Excel to verify the above calculations.*

iv.

PROBLEM 6. *Use Excel for this question. From Section 6.2 (5th edition), do problem 14. Also answer these questions:*

A. What is the probability that **exactly** *3/4 of the men are wearing their ties too tight?*

B. What is the probability that **at least** *3/4 of the men are wearing their ties too tight?*

(b) Key formula

$P(r \text{ successes }) = C_{n,r}p^r q^{n-r}$

Along with the rest of probability theory! (complements, multiplication rule, addition rule, etc)

PROBLEM 7. *Use the formula to calculate some of the above probabilities by hand.*

PROBLEM 8. *From Section 6.2, do problem 15. Do it by hand, and using Excel.*

(c) Expectation of binomial distribution

Expectation = np

PROBLEM 9. *Steve Nash has the highest career foul shooting percentage of* 90.4% *(stats based on the last time I checked ...).*

i. *If he shoots 100 foul shots, how many do we expect to go in?*

ii. *If he shoots 25 foul shots, how many do we expect to go in?*

iii. *Do a simulation of 25 foul shots in Excel using RAND, to see how many go in. How close is the simulation to the expected value?*

(d) Standard Deviation, $\sigma = \sqrt{npq}$