CSI 35 LECTURE NOTES (Ojakian)

Topic 10: More Introductory Graph Theory

OUTLINE

(References: Wells: 152 - 156, Rosen: 10.2, 10.3, 10.4)

1. Representing Graphs on Computer
2. Subgraphs, Isomorphism, Connectivity

---

1. Computer Graph Representation

(a) Draw basic example on board for both below.

(b) Vertex list representation

PROBLEM 1. *Enter a graph into Python using list representation and plot it. Also list and count the number of edges.*

(c) Matrix Representation (of simple graphs) by adjacency matrix.

One of the problems in Good Will Hunting!

PROBLEM 2. *Get the adjacency matrix of the last graph using Python.*

PROBLEM 3. *Enter a graph into Python using adjacency matrix and plot it.*

2. Subgraphs

(a) In general in math: Consider subobjects of an object (ex: subset of set)

(b) Subgraphs and Induced Subgraphs

3. Isomorphism

(a) Definition. Via functions. Visualize as relabeling.

(b) Invariants: Number of vertices and edges, degree sequence, etc.

PROBLEM 4. *What are other invariants?*

(c)

PROBLEM 5. *Wells Exercise 155.1.4.*
And try some graphs in Sage.

4. Connectivity Issues

How reliable is the network?

(a) Connected or not?

(b) If disconnected: What are the connected components?

(c) Define vertex connectivity (κ). Another graph invariant.

i. Examples: Cycles, Trees, Complete Graphs

5. An application and a simulation

Small world phenomena (6 degrees of separation ...).

PROBLEM 6. *Create a random graph that simulates the world and its connections, and see what the degree of separation is.*

6. Exercises

(a) Section 10.2. Subgraphs. 35a, 36, 50 - 53

(b) Section 10.2. Degree Sequences. 38, 39 (not e), 40 - 43.

(c) Section 10.3. Adjacencey list and matrix. 1 - 15, 29, 50

(d) Section 10.3. Isomorphism. 38 - 48, 49, 58 - 63