

CSI 35 LECTURE NOTES (Ojakian)

Topic 9: Introduction to Graph Theory

OUTLINE

(References: Wells: 152 - 156, Rosen: 10.1, 10.2)

1. Basic terminology of graph theory
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1. Graph Definition (10.1)

- (a) Definition: Simple Graph
 - i. Example 1: Friendship Network or Acquaintanceship Graph
 - ii. Vertices = Nodes
 - iii. Edges give “adjacency”
- (b) Definition: Multi-Graphs
 - i. Example 10: Road Network
- (c) Definition: Directed Graphs
 - i. Edges = Arcs
 - ii. Example 5: Web Graph
 - iii. Example 8: Precedence Graph for programs.

2. Graph Terminology (10.2)

- (a) Connected versus Disconnected
- (b) Neighbor and neighborhood (i.e. “open” neighborhood)
- (c) Degree of vertex (and out-degree and in-degree in directed graph)
- (d) Handshaking Lemma

PROBLEM 1. *Demonstrate and prove the Handshaking Lemma.*

PROBLEM 2. *In any simple graph, how many odd degree vertices are there?*

PROBLEM 3. *What is the similar lemma for directed graphs, that connects out-degree and indegree to the number of edges? Prove it.*

3. Some Special Graphs

- (a) Cycle (C_n)
- (b) Path (P_n)
- (c) Complete (K_n)
- (d) Complete bipartite ($K_{a,b}$)
- (e) Wheel (W_n). Note has $n + 1$ vertices.

PROBLEM 4. *How many edges are in K_n ? (exercise 481 from Finan) Give a proof by induction. Also can prove by combinations.*

- (f) Bipartite

PROBLEM 5. *Wells Exercise 153.3.3 (identifying bipartite graphs).*

- (g) Tree (connected and acyclic). We'll return to.

4. Exercises

- (a) Section 10.1: 7- 9. Ignore instructions in book. Instead:
 - i. How many arcs are there?
 - ii. What is the indegree and outdegree of each vertex?
- (b) Section 10.2: 20 (not f), 23, 24, 30, 35
- (c) Section 10.1: 33 (marriage graph). How change for same-sex marriages also included?
- (d) Section 10.2: 1 - 10, 12, 17, 18*, 37
- (e) Section 10.2: 19 - a bet you're guaranteed to win! ...
- (f) Section 10.2. Bipartite. 21 - 25, 26 (not d)