

CSI 35 LECTURE NOTES (Ojakian)

Topic 7: Equivalence Relations

OUTLINE

(References: Wells 51 - 53, 55 - 59, 117, 129, 130, 132 - 137; Rosen: 9.5)

1. Equivalence Relations
 2. Partitions
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1. Equivalence Relations

- (a) Intuitive idea: Want to ignore some aspects of the situation, and say that objects are equivalent for our purposes if certain conditions are met.
- (b) Less mathy example: Card game where cards are equivalent if same color (say for some game)
- (c) Everyday example: kids at school are equivalent if they are the same age (say for arranging classes)
- (d) Example 4 (section 9.5, p. 640). If just care about length. Check next properties of def.
- (e) DEF: Binary relation which is reflexive, symmetric, and transitive.
- (f)

PROBLEM 1. Find 2 different equivalence relations on the set $\{a, b, c\}$.

- (g) Example 7 (section 9.5, p.641). Non-equivalence relation $|x - y| < 1$
- (h) Exercise 7 (section 9.5, p.646). Equivalence relation of truth.
- (i)

PROBLEM 2 (Checking if equivalence relation). Do Wells 129.2.2 and 129.2.9 (all of 129.1 to 129.9 good)

PROBLEM 3. Prove that for any n , congruence mod n , is an equivalence relation.

2. Partitions

- (a) Example: Wells 117.1.13.
- (b) Do Wells 117.1.9.
- (c) Example: Congruence mod n partition.

3. Equivalence relations versus Partitions

Example: Congruence mod n .

- (a) Equivalence Class of an element x : $[x]$.
 - i. Fact: If x and y are in the same equivalence class, then $[x] = [y]$.
- (b) From Equivalence Relation to Partition: via Quotient Sets.
 - i. Example: Wells 132.2.2
- (c) From Partition to Equivalence Relations: being in the same piece.
 - i. Example: Take any finite equivalence relation.
 - ii. Example: Use the partition that we got in Wells 132.2.2.
- (d) Theorem 2 (Section 9.5, page 645): The fundamental theorem relating Partitions to Equivalence Relations.

4. More examples

- (a) Using equivalence to capture desired accuracy:
Equivalence relation on R , making two numbers equivalent if their decimal expansions agree down to and including the 1/100s place.
- (b) Make up one: Choose parameters for people, so each person is a list of attributes. Then choose what you care about and what you don't care about.

5. Exercises

- (a) Checking digraphs. Section 9.5: 21 - 23
- (b) Checking Matrices. Section 9.5: 24
- (c) Checking basic properties. Section 9.5: 1 - 7, 11 - 16, 25
- (d) Equivalence Classes. Section 9.5: 17, 18,
26 - 28 (based on 1 - 3),
29 (based on 25),
30 (based on 11),
31 (uses 12, 30),
32 (uses 13, 30)
35 - 37 (modular numbers)
- (e) Partitions. Section 9.5: 41 - 45
- (f) Partitions and Equiv Relations. Section 9.5: 47, 48