CSI 35 LECTURE NOTES (Ojakian)

Topic 6: Relations

OUTLINE

(References: Wells 51 - 53, 55 - 59, 117, 129, 130, 132 - 137; Rosen: 9.1, 9.2, 9.3)

1. Relations: Definitions and Examples
2. Database Application
3. Properties: Reflexivie, Anti-reflexive, Symmetric, Anti-symmetric, Transitive

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1. Relations
   (a) Binary and n-ary
      i. Subset of Cartesian product.
      ii. Notations: $(a, b) \in R$, or $R(a, b)$, or aRb.
      iii. Do some small finite examples.
   (b) Some more involved examples.
      i. $<$ on the rationals
      ii. Congruence mod n
      iii. (x, x^2) and (x^2, x), for x real.
      iv. $\{(x, y, z, n) \mid x, y, z, n$ are positive integers and $x^n + y^n = z^n\}$
2. Representing Relations (mostly binary ones)
   (a) Table or listing
   (b) Graph "on the plane"
   (c) Two columns with arrows
   (d) Digraph
   (e) Matrix
3. Viewed as databases

   Do in theory and sqlite.
   (a) Examples 11 and 12 (Section 9.2)
   (b) Have various "tables"
      i. Each column has a name and type
      ii. Each row is an item
      iii. Entire table can be viewed as n-ary relation
   (c) Create table
      i. "CREATE TABLE [table] ([col1] [type1], [col2] [type2], ...)"
   (d) Build database with Insert command
      i. "INSERT INTO [table] VALUES (value1 , value2, value3 ...)"
   (e) Retreive data with Select command
      i. "SELECT [columns] FROM [table] WHERE columns = [...] ‘’

4. Typical Properties for Binary Relations on a Set

   (a) Definition: Binary relation on a set.
   (b) Reflexive (and anti-Reflexive)
      i. Example: $\leq$ and $<$
      ii. Prove reflexive.
         A. To prove NOT: Find ONE x in the set so that (x, x) is NOT in relation.
         B. Finite case: List all pairs (x, x).
         C. Finite case: Draw digraph and observe every vertex has a loop
         D. Finite case: Construct matrix and observe every diagonal entry is 1.
         E. Infinite case: Reason why (x, x) in relation for every x in the set.
      iii. Prove anti-reflexive.
         A. To prove NOT: Find ONE x in the set so that (x, x) IS in relation.
         B. Finite case: Observe that there are no pairs (x, x).
         C. Finite case: Draw digraph and observe no vertex has a loop.
         D. Finite case: Construct matrix and observe every diagonal entry is 0.
         E. Infinite case: Reason why (x, x) is NOT in relation for every x in the set.
   (c) Symmetric (and anti-Symmetric). Two ways to define anti-symmetric.
      i. Example: Congruence and $\leq$
      ii. Do Rosen 9.1, 49 (Mistaken Proof).
      iii. Prove Symmetric.
         A. To prove NOT: Find one PAIR (x, y) so that (x, y) is IN the relation, but (y, x) is NOT in relation.
         B. Finite case: List every (x, y) (where $x \neq y$) in the relation, and note that (y, x) is also in the relation.
         C. Finite case: Draw digraph and observe between every two vertices, either there are NO arcs, or arcs BOTH ways.
         D. Finite case: Construct matrix and observe that it is a symmetric matrix.
         E. Infinite case: Start with the assumption that (x, y) is in the relation, then prove that (y, x) is in the relation.
      iv. Prove Anti-Symmetric.
         A. To prove NOT: Find $x \neq y$ so that both (x, y) and (y, x) are in the relation.
         B. Finite case: List every (x, y) (where $x \neq y$) in the relation, and note that (y, x) is NOT in the relation for every pair.
         C. Finite case: Draw digraph and observe between every two vertices, either there are NO arcs, or exactly ONE arc.
         D. Finite case: Construct matrix and observe that for any entry (i, j) which is a 1, its mirror image (j, i) is a 0.
         E. Infinite case: Start with the assumption that (x, y) and (y, x) are in the relation, and prove that $x = y$.

(d) Transitive

  i. Example: $\leq$
  ii. Hasse Diagrams for Transitive relations
  iii. Note funny case. To be transitive, if (x, y) and (y, x) are in the relation, then (x, x) must be too.
  iv. Prove Transitive.
    A. To prove NOT: Find two pairs (x, y) and (y, z) that are IN the relation, but (x, z) is NOT in the relation.
    B. Finite case: List every PAIR of PAIRS (x, y) and (y, z) (i.e. "middle entry" is the same) in the relation, and note in each case, (x, z) is also in the relation.
    C. Finite case: Draw digraph to help listing out pairs ...
    D. Infinite case: Start with the assumption that (x, y) and (y, z) are in the relation, then prove that (x, z) is in the relation.

5. Examples for Reflexive, Symmetric, and Transitive

PROBLEM 1. *Do Wells 59.1.3*

PROBLEM 2. *Do Wells 55.1.9.*

PROBLEM 3. *Do Wells 56.1.4.*

PROBLEM 4. *Do Wells 59.1.4.*

PROBLEM 5. *Begin Wells 58.1.10 (finishing it will be a question on the next class work).*

6. More Exercises

(a) Section 9.1: 2, 3, 4, 6, 7, 10, 11, 12, 14, 15, 16, 17
(b) Primary Keys, etc. Section 9.2: 2, 4, 7, 8a, 9a
(c) Matrix Representation. Section 9.3: 1 - 10.
(d) Digraph Representation. Section 9.3: 22 - 32