

# CSI 35 LECTURE NOTES (Ojakian)

## Topic 5: Inductive Proofs about Recursion

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### OUTLINE

(References: Wells sections 105-107, 124, 125; Rosen: 5.3, 5.4, 5.5)

#### 1. Inductive Proofs about Recursion

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##### 1. Inductive Proofs about recursively defined functions

- (a) Two views.
  - i. Proving a recursive function has a certain property
  - ii. Proving a recursive programs works.
- (b) For  $F(0) = 1$  ;  $F(n) = 2 \cdot F(n - 1)$ : Prove that  $F(n) = 2^n$
- (c) Example of finding and solving (Wells 124.2.3): Compounding interest annually.
  - i. Let  $A(0) = P$  (the principal)
  - ii.  $A(n + 1) = \dots$
- (d) Do Wells exercise 107.3.12 (page 162); using total induction.
- (e) Prove  $C(n,k)$  formula (Thm 125.6) by induction (probably skip!)

##### 2. Structural Inductive Proofs

- (a) Form.
  - i. Prove the property is true of all elements specified in the basis step.
  - ii. Prove that the recursive step preserves the property (i.e. if the property holds of given elements, then it holds of an element constructed from these)
- (b) Recall divisibility rule: If  $a|x$  and  $a|y$ , then  $a|(x + y)$ .
- (c) Section 5.3, Example 10 (about set with 3 and closed under +), but just show: Every element of  $S$  is divisible by 3.
- (d) Section 5.3, Exercise 26. Then they do 27.
- (e)

**PROBLEM 1.** Define a subset of binary strings  $S$  as follows (implicit operation is concatenation):  $01 \in S$  and  $10 \in S$  and if  $x \in S$  then  $xy \in S$ .

- i. List all the length 4 and length 5 binary strings in  $S$
- ii. Prove that the strings in  $S$  all have an equal number of 0's and 1's.

##### 3. Exercises

- (a) Recursive Proofs.
  - Section 5.3: 12 - 16, 18.
  - Section 5.4: 16, 18,
- (b) Recursive Proofs on Sets. Section 5.3: 22, 26, 27, 28, 29, 32
- (c) Recursive proofs of program correctness. Section 5.5: 7.