CSI 35 LECTURE NOTES (Ojakian)

Topic 9: Introduction to Graphs

OUTLINE

(References: Wells: 152 - 156, Finan: Ch. 7, Rosen: 10.1, 10.2, 10.3)

1. Basic terminology of graph theory

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1. Graph Definitions and Applications

(a) Definition: Simple Graph (for example: Friendship Network)

i. Vertices = Nodes

ii. Edges

(b) Definition: Multi-Graphs (for example: network of roads)

(c) Definition: Directed Graphs (for example: Web links)

i. Edges = Arcs

(d) Adjacent, neighbor, and neighborhood.

(e) Vertex list representation

PROBLEM 1. *Enter a graph into SAGE using list representation and plot it. Also list and count the number of edges.*

(f) Matrix Representation (of simple graphs) by adjacency matrix.

PROBLEM 2. *Get the adjacency matrix of the last graph using Sage Math.*

PROBLEM 3. *Enter a graph into SAGE using adjacency matrix and plot it.*

2. Some Special Graphs

(a) Cycle (C_n)

(b) Path (P_n)

(c) Complete (K_n)

PROBLEM 4. *How many edges are in K_n? (exercise 481 from Finan) Give a proof by induction.*

(d) Bipartite

PROBLEM 5. *Wells Exercise 153.3.3 (identifying bipartite graphs).*

(e) Tree (connected and acyclic). We'll return to.

3. Basic Definitions

   (a) Subgraphs and Induced Subgraphs

   (b) Degree of vertex (and out-degree and in-degree in directed graph)

   (c) Handshaking Lemma

   PROBLEM 6. *Demonstrate (in Sage and/or by physically doing it) and prove the Handshaking Lemma.*

   PROBLEM 7. *In any simple graph, how many odd degree vertices are there?*

   PROBLEM 8. *What is the similar lemma for directed graphs, that connects out-degree and indegree to the number of edges? Prove it.*

4. Traveling through a graph

   Note: Due to variation in terminology, we will use decriptive names!

   (a) "Path"

   (b) "Path with no edge repeats"

   (c) "Path with no repeats"

   (d) Shortest Path (and distance and diameter)

5. Connectivity Issues

   (a) Connected

   (b) Connected Components

   (c) Recall Trees

6. Isomorphism

   (a) Definition

   (b) Invariants: Number of vertices and edges, degree sequence, etc.

   PROBLEM 9. *What are other invariants?*

   (c)

   PROBLEM 10. *Wells Exercise 155.1.4.*
   And try some graphs in Sage.

7. An application and a simulation

   Small world phenomena (6 degrees of separation ...).

   PROBLEM 11. *Create a random graph that simulates the world and its connections, and see what the degree of separation is.*