CSI 35 LECTURE NOTES (Ojakian)

Topic 8: Relations

OUTLINE

(References: Wells 51 - 53, 55 - 59, 117, 129, 130, 132 - 137; Rosen 9.1, 9.2, 9.3, 9.5, 9.6)

1. Relations: Definitions and Examples
2. Properties: Reflexivie, Symmetric, Transitive
3. Equivalence Relations and Partitions

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1. Relations
   (a) Binary and n-ary
      i. Subset of Cartesian product.
      ii. Notations: $(a, b) \in R$, or $R(a, b)$, or aRb.
      iii. Do some small finite examples.
   (b) Some more involved examples.
      i. $<$ on the rationals
      ii. Congruence mod n
      iii. (x, x^2) and (x^2, x), for x real.
      iv. $\{(x, y, z, n) \mid x, y, z, n$ are positive integers and $x^n + y^n = z^n\}$
2. Representing Relations (mostly binary ones)
   (a) Table or listing
   (b) Graph "on the plane"
   (c) Two columns with arrows
   (d) Digraph
3. Functions
   (a) Example: (x, x^2) versus (x^2, x) from above.
   (b) Definition of "functional relation" or a "relation which is a function".
   (c) The "Vertical Line Test".
   (d) Consider some small finite examples, some are, some are not.

4. <u>Typical Properties for Binary Relations on a Set</u>

   (a) Definition: Binary relation on a set.

   (b) Reflexive (and Irreflexive)

      i. Example: $\leq$ and $<$

   (c) Symmetric (and anti-Symmetric)

      i. Example: Congruence and $\leq$

      ii. Do Rosen 9.1, 49 (Mistaken Proof).

   (d) Transitive

      i. Example: $\leq$

      ii. Hasse Diagrams for Transitive relations

   (e) Note! - If you make up a "random" relation, it probably has none of the above properties.

5. <u>Examples for Reflexive, Symmetric, and Transitive</u>

   PROBLEM 1. *Do Wells 59.1.3*

   PROBLEM 2. *Do Wells 55.1.9.*

   PROBLEM 3. *Do Wells 56.1.4.*

   PROBLEM 4. *Do Wells 59.1.4.*

   PROBLEM 5. *Begin Wells 58.1.10 (finishing it will be a question on the next class work).*

   PROBLEM 6. *Consider the set of subsets of* $\{1, 2, 3\}$. *Which properties does it have? Note: This is a partial order. Draw a Hasse Diagram.*

6. <u>Equivalence Relations</u>

   Basic Examples: i) Equality. ii) Being the same color card.

   PROBLEM 7. *Find 2 different equivalence relations on the set* $\{a, b, c\}$.

   PROBLEM 8. *Do Wells 129.2.2 and 129.2.9.*

   PROBLEM 9. *Prove that for any* n, *congruence mod* n, *is an equivalence relation.*

7. <u>Partitions</u>

   (a) Example: Wells 117.1.13.

   (b) Do Wells 117.1.9.

   (c) Example: Congruence mod n partition.

8. Equivalence relations versus Partitions

   Example: Congruence mod n.

   (a) From Equivalence Relation to Partition: via Quotient Sets.

      i. Example: Wells 132.2.2

   (b) From Partition to Equivalence Relations: being in the same piece.

      i. Example: Take any finite equivalence relation.

      ii. Example: Use the partition that we got in Wells 132.2.2.

   (c) The fundamental theorem relating Partitions to Equivalence Relations.

9. More examples

   (a) Equivalence relation on strings such that xEy if x and y are the same length.

   (b) Equivalence relation on R, making two numbers equivalent if their decimal expansions agree down to and including the 1/100s place.

   (c) Equivalence relation on directed paths, taking two paths to be equivalent if the paths are the same length.

   (d) Make up one: Choose parameters for people, so each person is a list of attributes. Then choose what you care about and what you don't care about.