CSI 35 LECTURE NOTES (Ojakian)

Topic 6: Mathematical Induction

OUTLINE

(References: Wells 102-104, Rosen 5.1, 5.2)

1. Mathematical Induction
2. Strong Induction
3. Well-Ordering

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1. Summation Notation

   (a) Compute some examples.

   PROBLEM 1. *Use the summation notation to write the sum of the positive even integers from 2 up to and including 1000.*

2. First Induction Proof

   PROBLEM 2. *Prove that* $1 + 2 + \ldots + n = n(n+1)/2$

   (a) Base Step

   (b) Inductive Step

   (c) Toy example: The infinitely long subway route (show that the subway stops at all the stations).

3. More Inductive Proofs

   (a) Prove that $n^3 - n$ is divisible by 3, when n is a positive integer (Section 5.1, Example 8, from Rosen).

   (b) Do exercise 19 from section 5.1 of Rosen, but first true to prove < 2 "directly."

   (c) Prove that a set with n elements has 2^n subsets (Section 5.1, Example 10, from Rosen).

   (d) Let n be a positive integer. Show tht every 2^n by 2^n checkerboard with one square removed (anywhere) can be tiled using "right triominoes." (Section 5.1, Example 14, from Rosen)

4. Mistaken Inductive Proofs

   (a) Rosen, exercises - 49 and Example 15.

5. Strong Induction

   (a) Introduction

   PROBLEM 3. *Do exercise 1, section 5.2, from Rosen (p. 341).*

   PROBLEM 4. *Prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps (Example 4 from Rosen, 5.2, page 337; note: can do with usual induction too).*

   (b) Mistakes in Strong Induction

   PROBLEM 5. *Do Rosen, section 5.2, exercise 29 (p. 343).*

   (c) Games!

      i. 2-player, no-chance, zero-sum, perfect information.
      ii. **Theorem**: For such games, exactly one player has a winning strategy.
      iii. Example: Tic-Tac-Toe does not have a winning strategy!
      iv. The game of Nim. Play it online. Describe winning strategies and give proofs.
      v. The game of Chomp. Play it online. Winning strategies?
         A. Describe Strategy-Stealing approach.
         B. Do Rosen, section 5.2, exercise 15.

6. Well-ordering and why induction works?

   (a) Smallest counter-example for natural numbers. For integers? For reals?

   (b) Consider: Well-ordering principle $\Rightarrow$ Induction Principle.