CSI 35 LECTURE NOTES (Ojakian)

Topic 5: Proof Techniques

OUTLINE

(References: Wells 15, 27, 28, 30, 80-84, 86; Rosen 1.7, 1.8)

1. Inference Rules
2. Proof by Contraposition and Contradiction

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1. Rules of Inference
   (a) Turnstyle: Assumptions $\vdash$ Conclusions
   (b) Definition leads to inference rule. Pick example!
   (c) Theorem leads to inference rule. Pick example!
   (d) Any true implication leads to an inference rule. Pick example!
   (e) Modus Ponens
   (f) Direct Proof - What we have been doing up till now.

   PROBLEM 1. *Give a structured proof of the following: The product of any two consecutive integers is even.*
   And for each step of the proof write down an inference rule used to arrive at that step (they can be a bit rough/vague).

2. Proof by Contraposition and Contradiction
   (a) Note propositional logic equivalence to $P \Rightarrow Q$.

   PROBLEM 2. *Prove that for all positive integers n, if n^2 is even then so is n. (Use contraposition)*

   PROBLEM 3. *Prove that among 100 consecutive days, any 51 of these days must contain 2 consecutive days.*
   (Use contraposition)

   (b) Proof by contradiction.

   PROBLEM 4. *Do Problems 2 and 3 by contradiction.*

3. Proving Equivalences

   Must prove two implications.

   PROBLEM 5. *Prove that for all positive integers n, n^2 is even if and only if n is even.*
   (Note: part of this proof already done)

4. Some more proofs

   PROBLEM 6. *Can we tile the standard checkerboard using dominoes? (i.e. a dominoe covers 2 adjacent squares, vertical or horizontal) Give a proof of your answer.*

   PROBLEM 7. *Can we tile the standard checkerboard using dominoes, if one of the 4 corners is removed? Give a proof of your answer.*

   PROBLEM 8. *Can we tile the standard checkerboard using dominoes, if two diagonally opposite corners are removed? Give a proof of your answer.*