CSI 35 LECTURE NOTES (Ojakian)

Topic 3: Introduction to Proofs

OUTLINE

(References: Wells 5)

1. Structured Proofs

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1. <u>Structured Proofs</u>

   PROBLEM 1. *Prove that every integer divides itself.*

   (before trying to prove it, play with examples, and get a feel for the statement)

   I will use the expression "**Structured Proof**" to refer to a proof which is broken down into simple steps, in which the steps are successively numbered (i.e. 1, 2, 3, etc) and for each step a justifications is written down; the justification can be 1) by definition, or 2) by given, or 3) by a known fact, or 4) by indicating how earlier steps logically imply this step (include the numbers of the steps you use).

   In (3) when I say "known fact" I do *not* mean facts you think are simple about the new definition (ex. you can't just assume things about divisibility). I mean facts from *earlier courses* (ex. distributivity, basic algebra, facts about integers, etc)

   PROBLEM 2. *Give a structured proof that every integer divides itself.*

   PROBLEM 3. *Prove that 103 divides itself.*

   (in the last problem, can use "Universal Instantiation")

2. <u>More Proofs</u>

   PROBLEM 4. *For each of the following propositions, is it true or false; justify your answers.*

   (a) Every integer divides 0.

   (b) 0 divides every integer.

   PROBLEM 5. *Consider two positive integers that divide one another; what can you say about them? Test your hypothesis with a program. Prove it (avoid "existential bigamy"!).*

   PROBLEM 6. *Prove that if m and n are both perfect squares, then nm is also a perfect square.*

   (we'll need to understand the definition of **perfect square** *to do this problem)*

3. Proofs on program correctness

(a) One kind of question: Does the program always terminate?

(b) Second kind of question: Given some initial assertion, verify some final assertion.

PROBLEM 7. *Consider the following program:*

```
y = 5
if y <= 5:
    z = x + 2*y
else:
    z = x - 2*y
```

Suppose the initial assertion is that $x = 3$*. Then prove the final assertion that* $z = 13$*.*

PROBLEM 8. *Consider the following program:*

```
x = 0
for k in range(1, n+1):
    x = x + 2*k
```

Suppose the initial assertion is that $n = 100$*. Then prove the final assertion that* $x = 10100$*.*

PROBLEM 9. *Consider the following program:*

```
while x < 10000:
    if x % 7 == 0:
        x = x + 1
    else:
        x = x + 3
```

Prove that whatever integer the initial value of x *is, the program terminates.*