Kerry Ojakian's CSI 35 Class
Due Date: TBA

HW #3

General Instructions: Homework is to be handed in at the beginning of class. While you may work with others from class, you may not copy. For details on working with others, see the Class Contract (available at the web page), which you have signed.

The Assignment

1. From Rosen, section 5.1, do exercise 50 (p. 331).
2. Suppose you want to prove the following:

   $$1/2 + 1/4 + 1/8 + \ldots + 1/2^n < 1$$

   Try to prove this "directly," using induction. I assume your attempt will fail. Describe the difficulty you run into. Now try another approach:

   (a) By experimenting with small values of n, guess an exact formula for the sum.
   (b) Prove that your guess is true.
   (c) As a corollary conclude: $1/2 + 1/4 + 1/8 + \ldots + 1/2^n < 1$
3. Suppose n is and integer and $n \geq 2$. Prove that a set with n elements has $n(n-1)/2$ subsets of size exactly 2.
4. From Wells, do exercise 125.6.10 (p.195).
5. From Wells do exercise 107.3.13 (p. 163). Prove using induction.

   Hint: On part (b) unfold the recursion multiple times and use strong induction
6. Rosen, section 5.2, exercise 4 (p. 341-342).
7. Rosen, section 5.2, exercise 30 (p. 344).
8. Rosen, section 5.2, exercise 16 (p. 342). Also, after proving that the first player has a winning strategy, actually describe the strategy (the strategy should be apparent from a correct inductive proof).
9. Consider the function $F(m, x) = mx$, for non-negative integers m and x. Give a recursive definition of this function which only uses addition. Write your function as a Python program in SageMath and save it to Dropbox.
10. Pick one of the proofs from the above problems and write a program in SathMath which gives good evidence of its truth. Explain how your program does this. Save the program to SathMath.