HW #1

General Instructions: Homework is to be handed in at the beginning of class. While you may work with others from class, you may not copy. For details on working with others, see the Class Contract (available at the web page), which you have signed.

The Assignment

1. Consider the following program:

   ```
   y = k
   for x in range(-k, 0):
       y = y - 1
   ```

   (a) Prove that if the initial assertion is that $k = 100$, then the final assertion of $y = 0$ is true.

   (b) Run the program in SageMath to see that this is the case, saving your sage work sheet to Dropbox.

   (c) Suppose k can initially be any integer. Come up with a theorem that states what the final values of y is, depending on the initial value of k. Prove your theorem.

2. From Wells, do Exercises 5.3.1 (p.7) and 5.5.4 (p.9)

3. For this problem, use the following definition of *odd*:

   An integer n is *odd* if there is an integer k such that $n = 2k + 1$.

   Prove that the product of two odd integers is odd.

4. A number is *Bronx* if it is an odd positive integer which 5 divides. Do the following:

   (a) Prove that 55 is Bronx.

   (b) Prove that 60 and -55 are not Bronx.

   (c) Prove that if a and b are both Bronx, then ab is also Bronx.

5. Find a statement of propositional logic which is neither a tautology nor a contradication. Use a truth table to justify your answer.

6. Find a predicate of one variable that is true of all integers.

7. Wells 75.2.3 and 75.3.3. Pick one of the problems and write a program in SathMath to give evidence its truth for a large and representative sample of integers; put your sage sheet in Dropbox.

8. Wells 79.1.6; only do part (d) if you like ...

9. Consider the following conjecture:

   Every even integer larger than 2 can be written as the sum of two primes.

   For example, the conjecture is true for 4 (since $4 = 2 + 2$), and true for 6 (since $6 = 3 + 3$), and true for 8 (since $8 = 3 + 5$).

   Write a program in SageMath to test this conjecture; indicate how you use your program to test the conjecture. Put your program in Dropbox.

   Do you believe the conjecture is true or false? Can you prove your answer?? (You don't have to attempt the proof ...)

10. Read the following selections from the Krantz article: Sections 1, 2, and 3 (pages 1 - 9), and section 9.1 (pages 32 - 33). Then, based on the reading, answer the following questions (**type your answers!** and attach here):

    (a) What is a mathematical proof? What are the various properties of a mathematical proof? What is some of the history of mathematical proof?

    (b) How does the reasoning process in mathematics differ from the reasoning process in science?