Postfix Expression

Postfix Expression

- Infix expression is the form AOB
  - A and B are numbers or also infix expression
  - O is operator (+, -, *, /)
- Postfix expression is the form ABO
  - A and B are numbers or also postfix expression
  - O is operator (+, -, *, /)

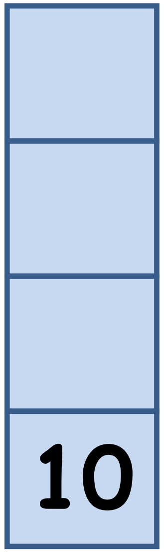
From Postfix to Answer

- The reason to convert infix to postfix expression is that we can compute the answer of postfix expression easier by using a stack.

From Postfix to Answer

Ex: 10 2 8 * + 3 -

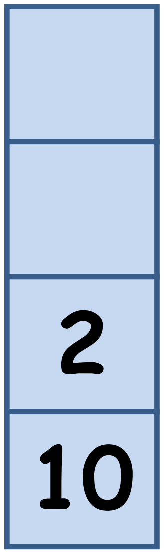
- First, push(10) into the stack



From Postfix to Answer

Ex: 10 2 8 * + 3 -

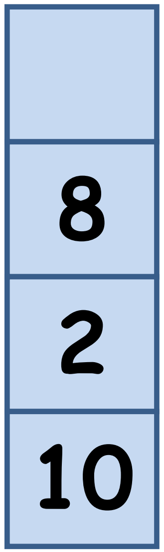
- Then, push(2) into the stack



From Postfix to Answer

Ex: 10 2 8 * + 3 -

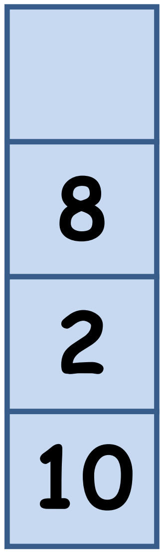
- Push(8) into the stack



From Postfix to Answer

Ex: 10 2 8 * + 3 -

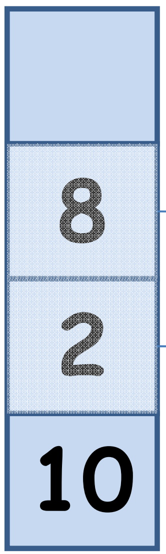
- Now we see an operator *, that means we can get an new number by calculation



From Postfix to Answer

Ex: 10 2 8 * + 3 -

- Now we see an operator *, that means we can get an new number by calculation
- Pop the first two numbers

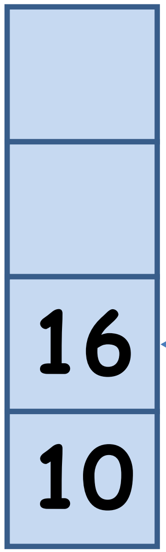


2 * 8 = 16

From Postfix to Answer

Ex: 10 2 8 * + 3 -

- Now we see an operator *, that means we can get an new number by calculation
- Push the new number back

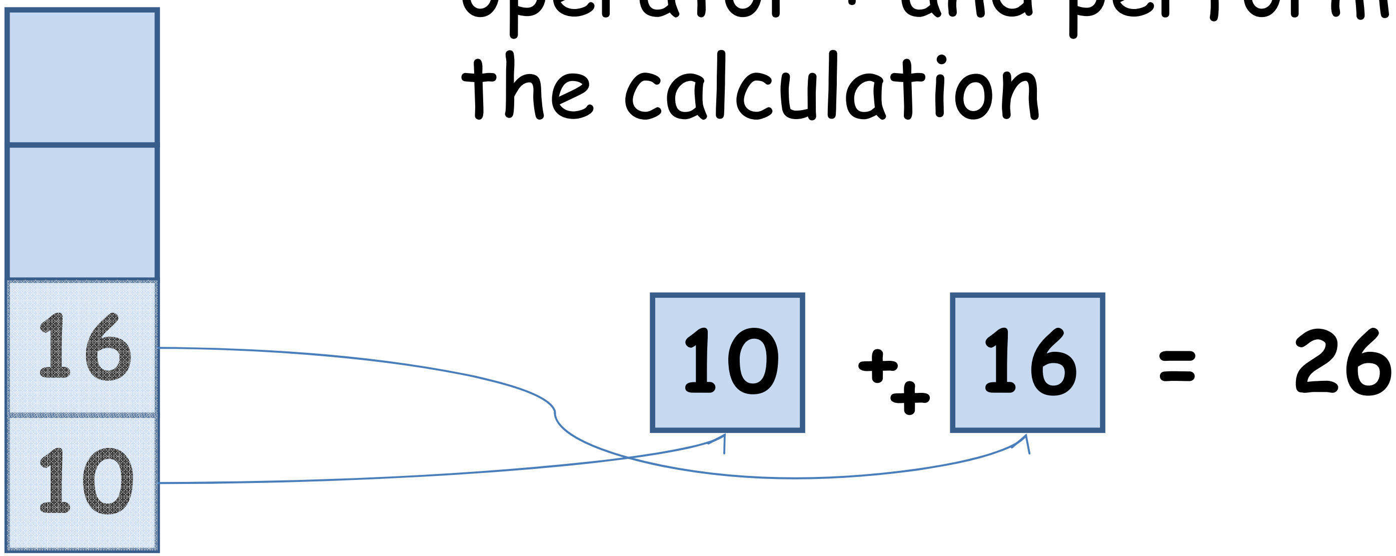


$\boxed{2} * \boxed{8} = 16$

From Postfix to Answer

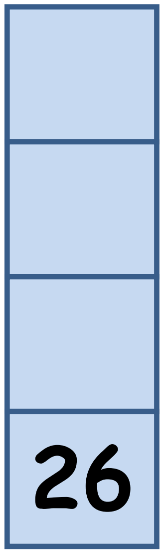
Ex: 10 2 8 * + 3 -

- Then we see the next operator + and perform the calculation



From Postfix to Answer

Ex: 10 2 8 * + 3 -



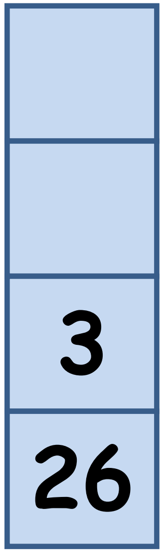
- Then we see the next operator + and perform the calculation
- Push the new number back

$$\boxed{10} + \boxed{16} = 26$$

From Postfix to Answer

Ex: 10 2 8 * + 3 -

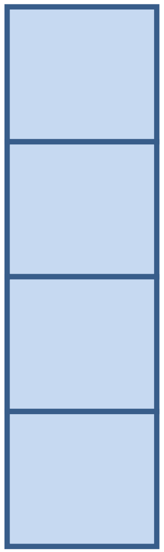
- We see the next number 3
- Push (3) into the stack



Compute the Answer

Ex: 10 2 8 * + 3 -

- The last operation

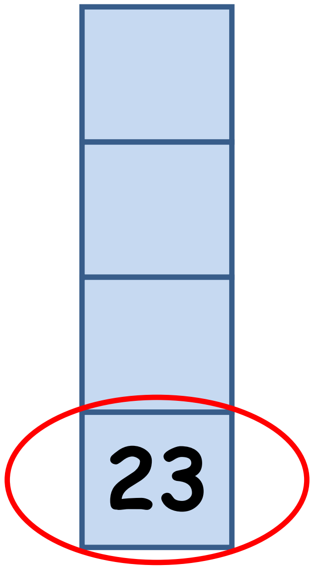


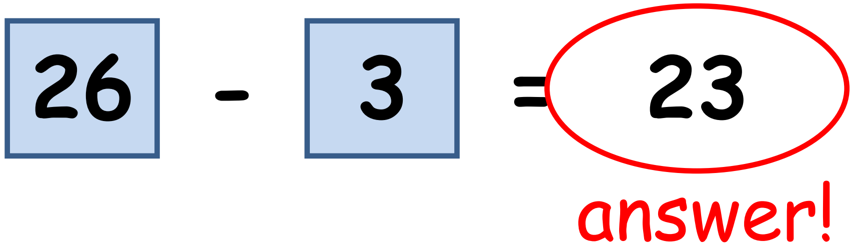
$\boxed{26} - \boxed{3} = 23$

From Postfix to Answer

Ex: 10 2 8 * + 3 -

- The last operation





From Postfix to Answer

- Algorithm: maintain a stack and scan the postfix expression from left to right
  - If the element is a number, push it into the stack
  - If the element is a operator O, pop twice and get A and B respectively. Calculate BOA and push it back to the stack
  - When the expression is ended, the number in the stack is the final answer

Transform Infix to Postfix

- Now, we have to design an algorithm to transform infix expression to postfix

Transform Infix to Postfix

- Observation 1: The order of computation depends on the order of operators
  - The parentheses must be added according to the priority of operations.
  - The priority of operator * and / is higher then those of operation + and –
  - If there are more than one equal-priority operators, we assume that the left one's priority is higher than the right one's
    - This is called left-to-right parsing.

Transform Infix to Postfix

- Observation 1: The order of computation depends on the order of operators (cont.)
  - For example, to add parentheses for the expression 10 + 2 * 8 - 3,
  - we first add parenthesis to 2 * 8 since its priority is highest in the expression.
  - Then we add parenthesis to 10 + (2 * 8) since the priorities of + and – are equal, and + is on the left of -.
  - Finally, we add parenthesis to all the expression and get ((10 + (2 * 8)) - 3).

Transform Infix to Postfix

- Observation 1: The order of computation depends on the order of operators (cont.)
  - The computation order of expression ((10 + (2 * 8)) - 3) is:
    - 2 * 8 = 16 → ((10 + 16) -3)
    - 10 + 16 = 26 → (26 – 3)
    - 26 – 3 = 23 → 23

Transform Infix to Postfix

- Simplify the problem, how if there are only +/- operators?

Transform Infix to Postfix

- Simplify the problem, how if there are only +/- operators?
- The leftmost operator will be done first
  - Ex: 10 - 2 + 3 → 8 + 3 → 11

Transform Infix to Postfix

- Simplify the problem, how if there are only +/- operators?
- Algorithm: maintain a stack and scan the postfix expression from left to right
  - When we get a number, output it
  - When we get an operator O, pop the top element in the stack if the stack is not empty and then push(O) into the stack

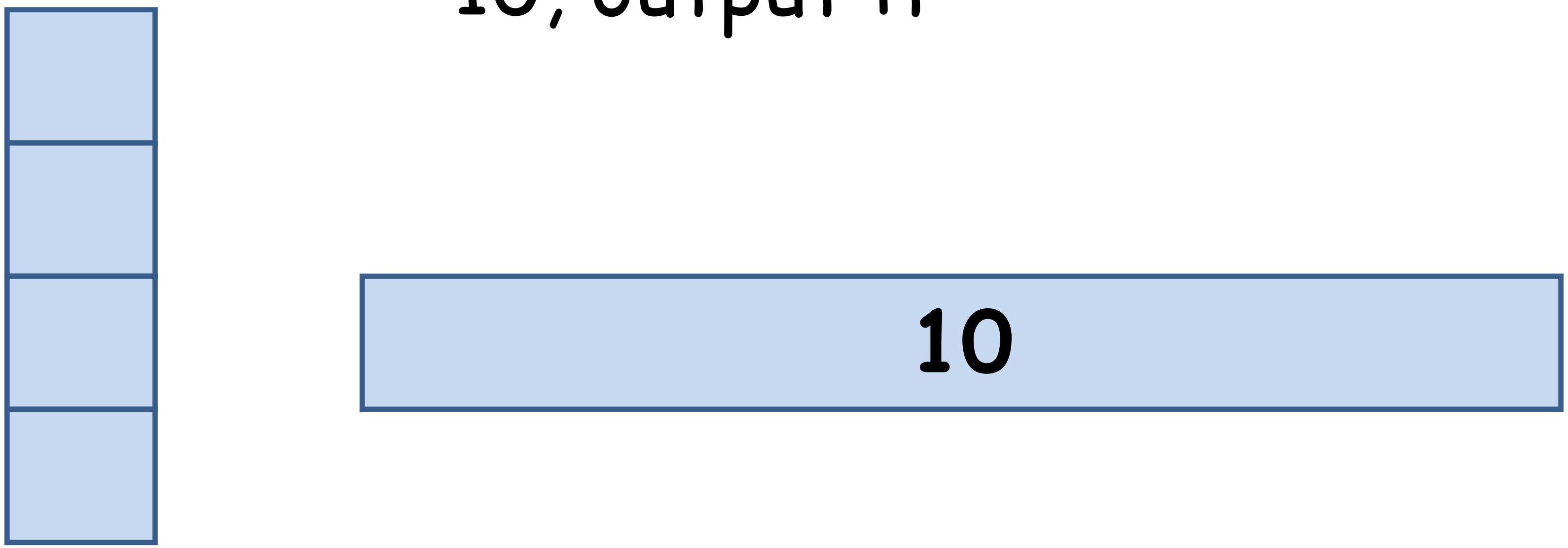
Transform Infix to Postfix

- Simplify the problem, how if there are only +/- operators?
- Algorithm: maintain a stack and scan the postfix expression from left to right
  - When we get a number, output it
  - When we get an operator O, pop the top element in the stack if the stack is not empty and then push(O) into the stack
  - When the expression is ended, pop all the operators remain in the stack

Transform Infix to Postfix

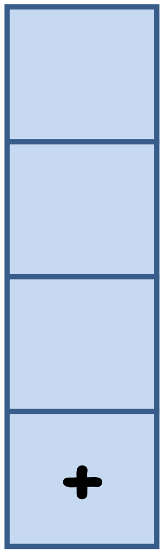
Ex: 10 + 2 - 8 + 3

- We see the first number 10, output it

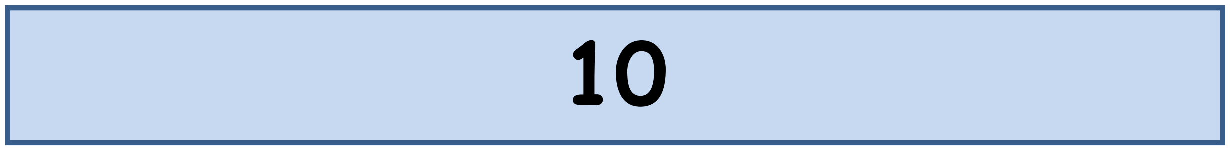


Transform Infix to Postfix

Ex: 10 + 2 - 8 + 3



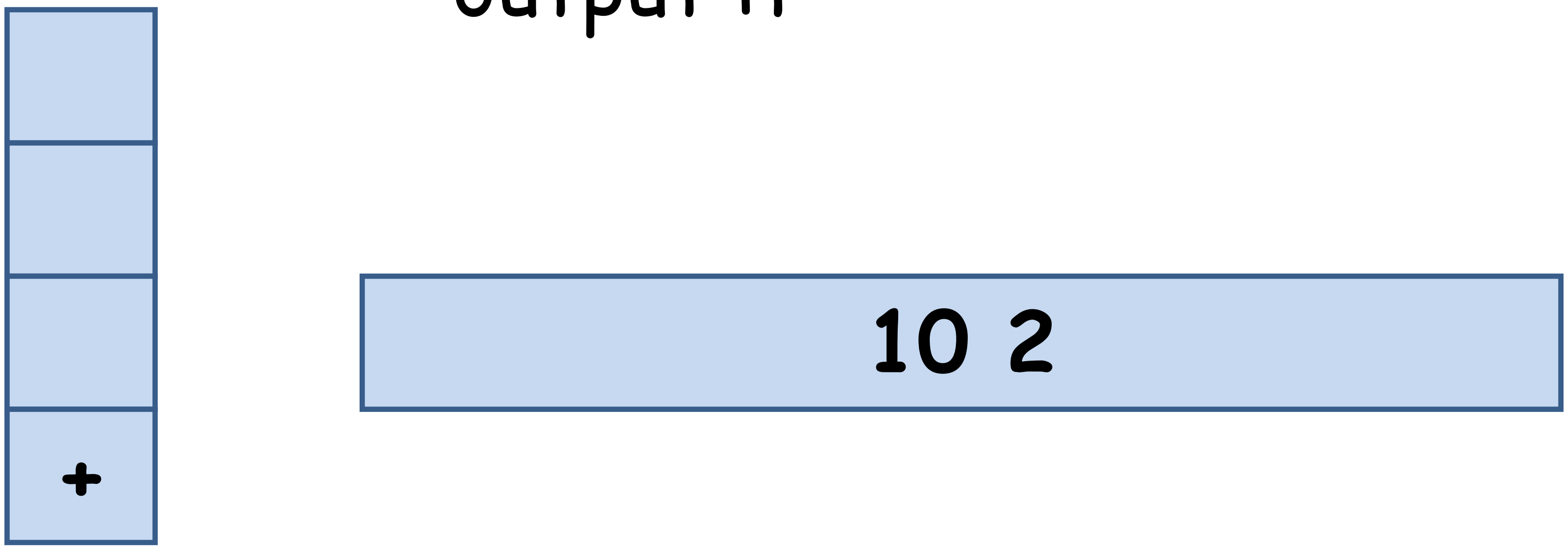
- We see the first operator +, push(+) into the stack because at this moment the stack is empty



Transform Infix to Postfix

Ex: 10 + 2 - 8 + 3

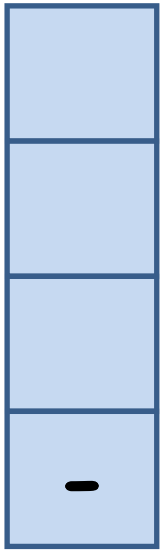
- We see the number 2, output it

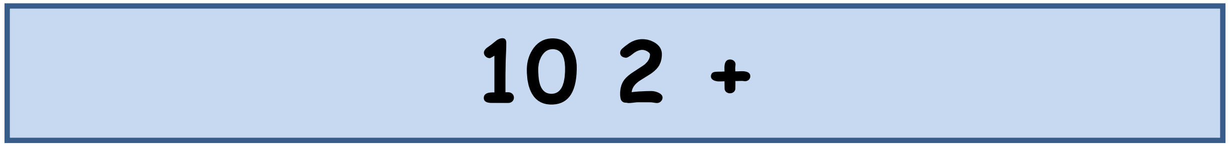


Transform Infix to Postfix

Ex: 10 + 2 - 8 + 3

- We see the operator -, pop the operator + and push(-) into the stack





Transform Infix to Postfix

Ex: 10 + 2 - 8 + 3

- We see the number 8, output it

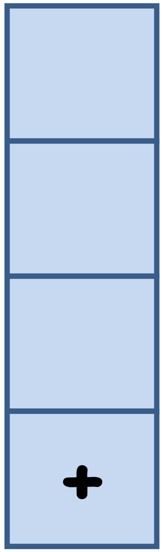
-

10 2 + 8

Transform Infix to Postfix

Ex: 10 + 2 - 8 + 3

- We see the operator +, pop the operator - and push(+) into the stack

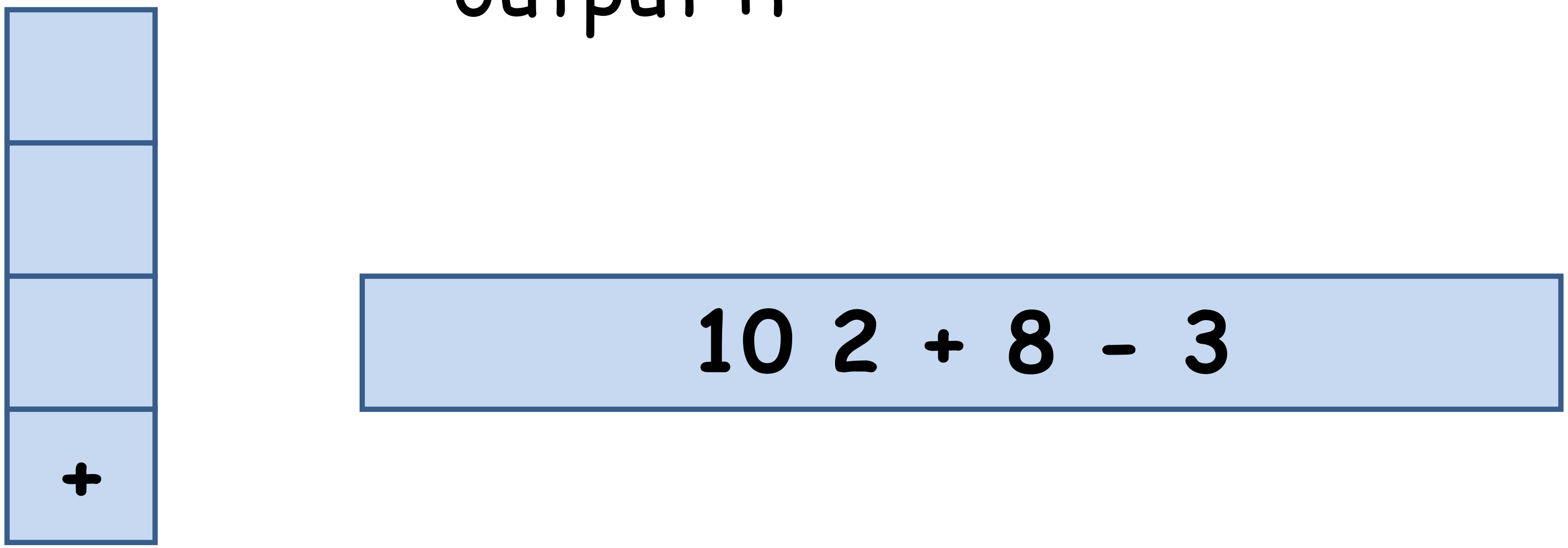


10 2 + 8 -

Transform Infix to Postfix

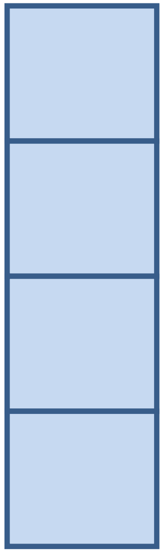
Ex: 10 + 2 - 8 + 3

- We see the number 3, output it



Transform Infix to Postfix

Ex: 10 + 2 - 8 + 3

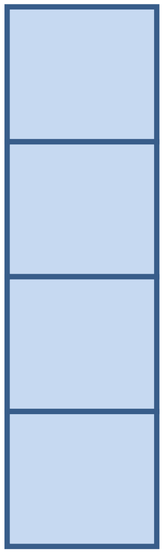


- We come to the end of the expression, then we pop all the operators in the stack

10 2 + 8 – 3 +

Transform Infix to Postfix

Ex: 10 + 2 - 8 + 3



- When we get an operator, we have to push it into the stack and pop it when we see the next operator.
- The reason is, we have to "wait" for the second operand of the operator

Transform Infix to Postfix

- How to solve the problem when there are operators +, -, *, / ?

Transform Infix to Postfix

- Observation 2: scan the infix expression from left to right, if we see higher-priority operator after lower-priority one, we know that the second operand of the lower-priority operator is an expression
  - Ex: a + b * c = a + (b * c) → a b c * +
  - That is, the expression b c * is the second operand of the operator "+"

Transform Infix to Postfix

- So, we modify the algorithm to adapt the situation

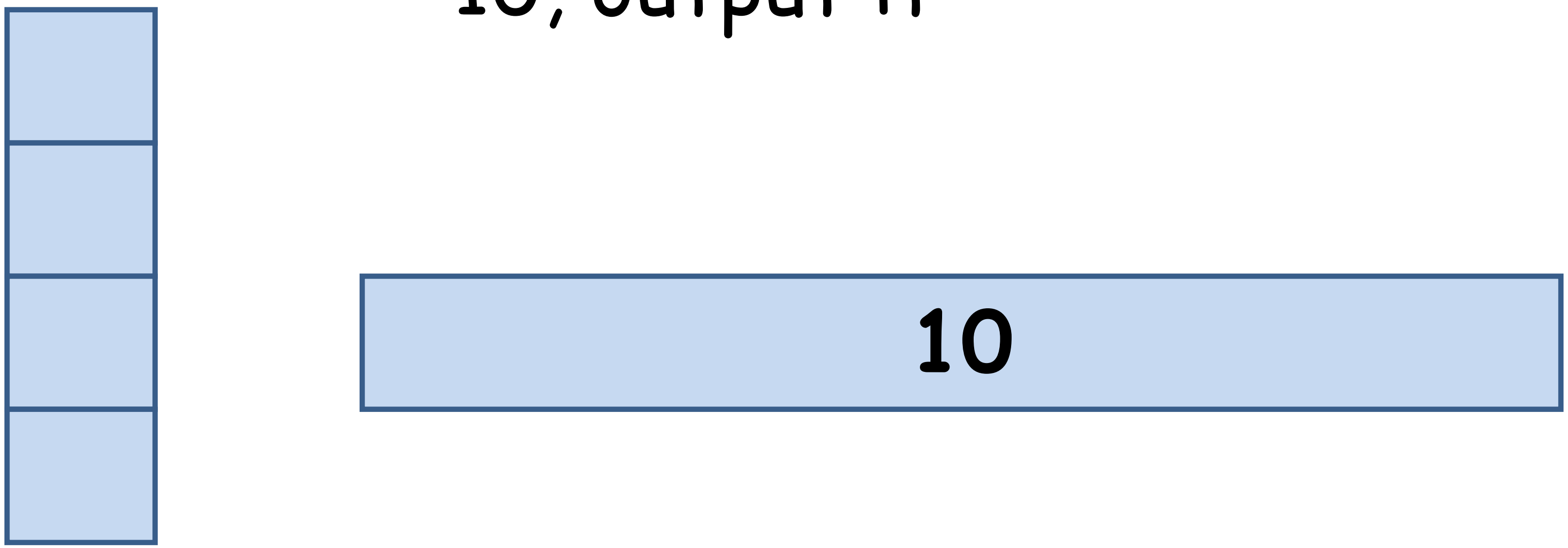
Transform Infix to Postfix

- Algorithm: maintain a stack and scan the postfix expression from left to right
  - When we get a number, output it
  - When we get an operator O, pop the top element in the stack until there is no operator having higher priority then O and then push(O) into the stack
  - When the expression is ended, pop all the operators remain in the stack

Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3

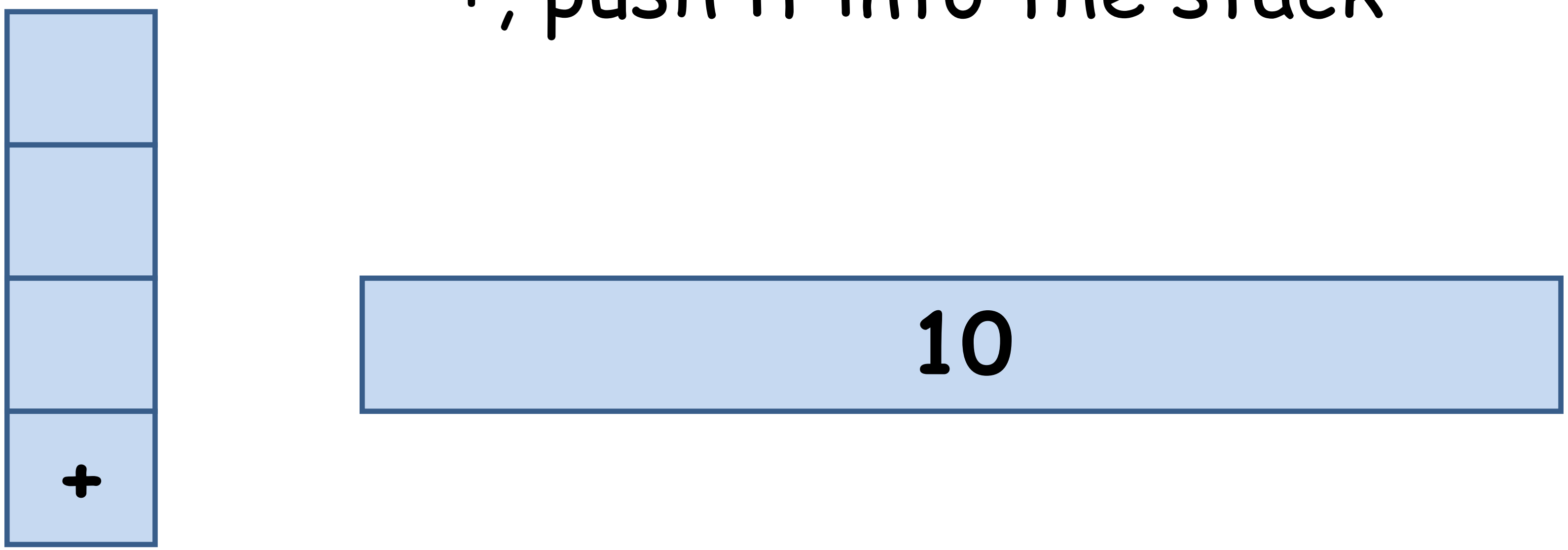
- We see the first number 10, output it



Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3

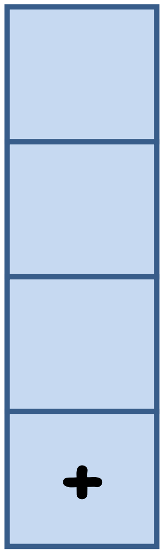
- We see the first operator +, push it into the stack

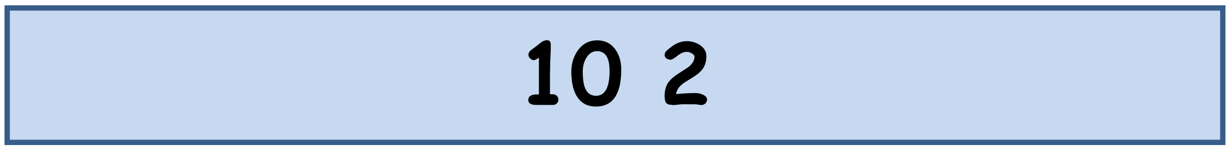


Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3

- We see the number 2, output it

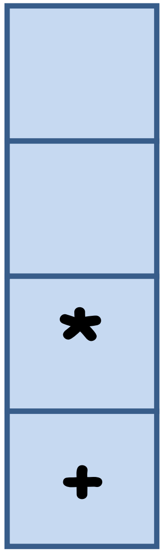


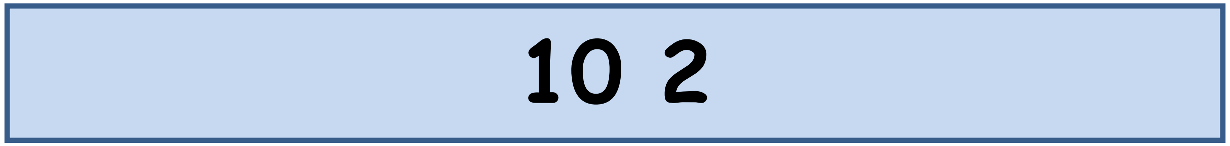


Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3

- We see the operator *, since the top operator in the stack, +, has lower priority then *, push(*)

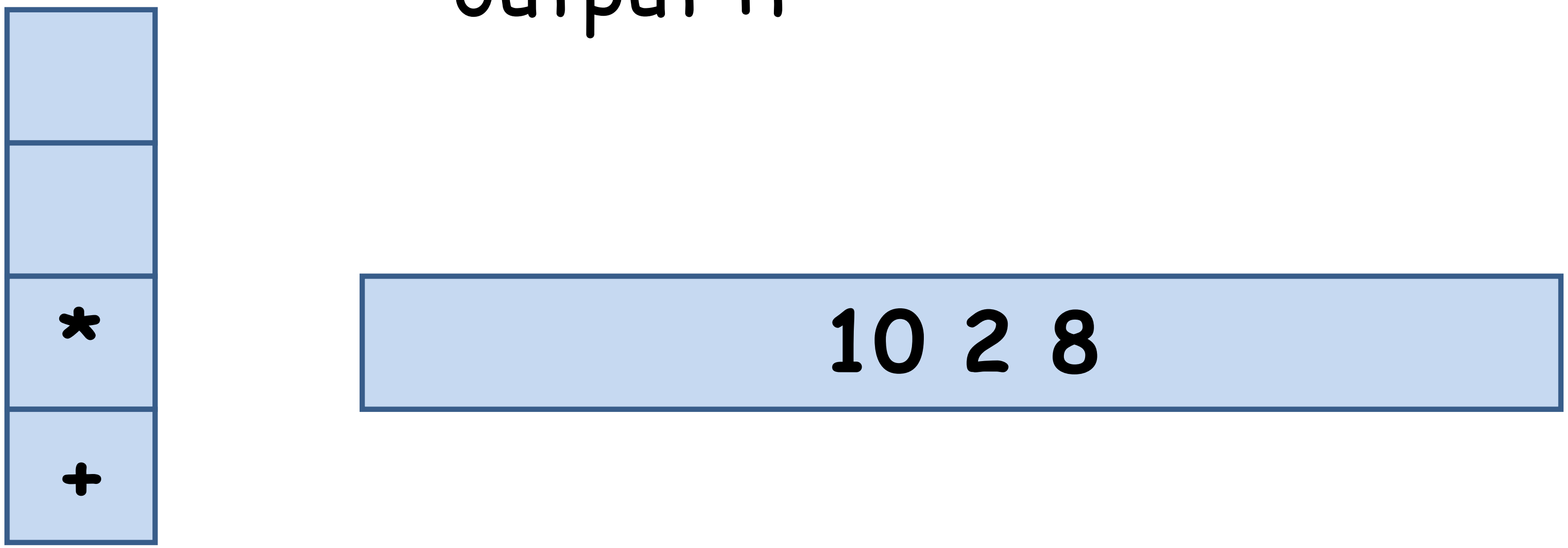




Transform Infix to Postfix

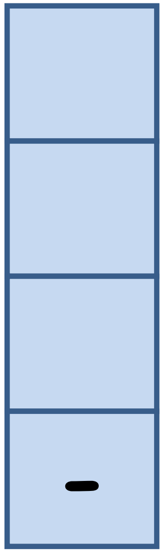
Ex: 10 + 2 * 8 - 3

- We see the number 8, output it



Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3



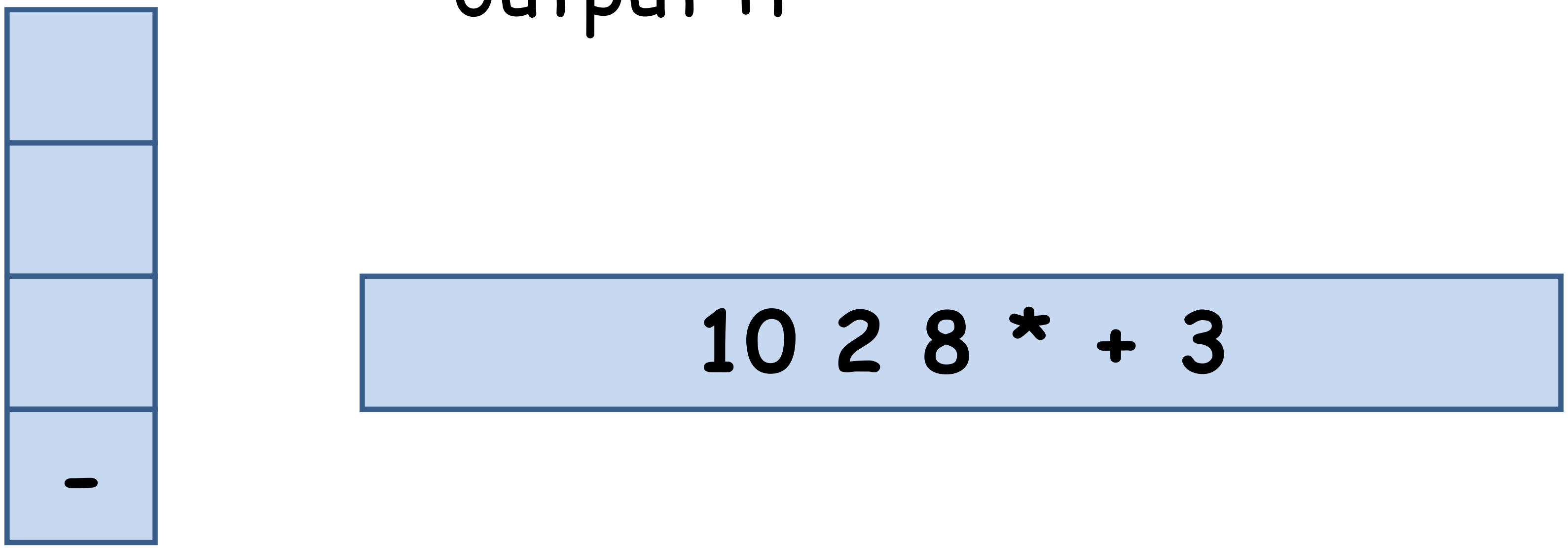
- We see the operator -, because its priority is lower then *, we pop. Also, because + is on the left of it, we pop +, too. Then we push(-)

10 2 8 * +

Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3

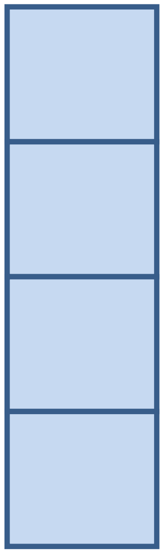
- We see the number 3, output it



Transform Infix to Postfix

Ex: 10 + 2 * 8 - 3

- Because the expression is ended, we pop all the operators in the stack



10 2 8 * + 3 -