CSI 30 LECTURE NOTES (Ojakian)

Topic 13: Proofs More Generally

OUTLINE
(References: 1.7, 1.8)

1. Mathematical Proofs

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1. Mathematical Proof

   (a) An argument that goes from the assumptions to the conclusion in a deductive manner.

   (b) Example: Consider summing two evens. What can you say? Can you prove it? Experiment with numbers.

   (c) The steps in a proof can be thought of as putting Inference Rules into action.

2. Structured Proofs

   (a) Example: Recall the evens - that was a structured proof.

   (b) Conjecture, counter-example, and proof

   (c) I will use the expression "**Structured Proof**" to refer to a proof which is broken down into simple steps, in which the steps are successively numbered (i.e. 1, 2, 3, etc) and for each step a justifications is written down; the justification can be 1) by definition, or 2) by given, or 3) by a known fact, or 4) by indicating how earlier steps logically imply this step (include the numbers of the steps you use).

   In (3) when I say "known fact" I do *not* mean facts you think are simple about the new definition (ex. you can't just assume things about divisibility). I mean facts from *earlier courses* (ex. distributivity, basic algebra, facts about integers, etc)

   (d)

   PROBLEM 1. *Prove that the sum of an odd integer and an even integer is odd.*

3. Using Cases in Proofs

   (a) Break up the statement you are trying to prove into a number of easier-to-prove statements

   (b) Examples

   Section 4.1: 9, 11, 12 (cases on a odd or even) and related 44, 45

   (c) Prove the 100 is not the cube of an integer.

   (d) Prove the triangle inequality: $|x + y| \leq |x| + |y|$ (for x, y integers); hint: Cases on the sign of x and y.

4. Proof by Contradiction

   To this point proofs have been Direct. Now Indirect or by contradiction.

   (a) Prove that if n is not prime, then at least one of its factors is $\leq n$.

   (b) Section 4.1: exercise 10

   PROBLEM 2. *Prove that for all positive integers n, if n^2 is even then so is n.*

   (c)

   PROBLEM 3. *Prove that among 100 consecutive days, any 51 of these days must contain 2 consecutive days.*

5. Proving Equivalences

   (a) Must prove two implications.

   PROBLEM 4. *Prove that for all positive integers n, n^2 is even if and only if n is even.*

   (Note: part of this proof already done)

   (b) The use of Lemmas:

      i. Lemma: The product of two evens is even.

      ii. Lemma: The product of two odds is odd