CSI 30 LECTURE NOTES (Ojakian)

Topic 7: Functions

OUTLINE
(References: 2.3)

1. Functions as kinds of relations

---

1. <u>Review Function Ideas Probably Seen</u>
   (a) Ways to present a function.
      i. Algebraic way
      ii. Table way
      iii. Graph way
   (b) Function Evaluation.
   (c) Vertical Line Test.
2. <u>Definition of Function</u>
   (a) Typical imprecise definition:
   A function f from set A to set B is an "assignment" of exactly one element of B to each element of A.
   (b) Precise definition using set products.
      i. Relation on $A \times B$.
      ii. Function definition:
      A relation on $A \times B$ that contains exactly one ordered pair (a, b) for every element $a \in A$, defines a function f from A to B.
   (c) Example represented in following ways
      i. List pairs (Class makes it up ...)
      ii. Table
      iii. Graph on the plane (and Vertical Line Test)
      iv. Two sides with lines/arrows
   (d) More Examples (which are functions?)
      i. All pairs of integers (x, y) such that $y = x^3 + 1$
      ii. $<$ on the rationals
      iii. (x, x^2) for x real
      iv. (x^2, x), for x real.

3. <u>Function Terminology</u>

   (a) Function = Map = Mapping = Transformation ...
   (b) Image = output = value
   (c) Domain
   (d) Codomain and Range
      i. Why codomain? Example: function $f : Z \to Z$ defined by $f(n) =$ [choose some polynomial]. So codomain clear, but range??

4. <u>Function Types</u>

   (a) Injective (or one-to-one)
      i. Horizontal Line Test
      ii. Showing that a function is *NOT* injective: Find counter-example (example: $f(x) = x^2$)
      iii. Showing *IS* injective: Go from $f(x) = f(y)$ to $x = y$.
   (b) Surjective (or onto)
      i. Showing that a function is *NOT* surjective: Find counter-example (example: $f : Z \to Z$ defined by $f(n) = 3n$)
      Showing that a function is *IS* surjective: From arbitrary y in codomain find x in the domain so that $f(x) = y$.
   (c) Bijective (or one-to-one correspondance)
      i. Example: $f : Even \to N$, $f(n) = n/2$ and cardinality significance.

5. <u>Building New Functions</u>

   (a) Function Composition (picture in Example 23).
      i. Do f then g: 2 notations - $g(f(x))$ or $g \circ f$
   (b) Inverse Function (exists iff bijective)
      i. f and g inverses iff $(f \circ g)(x) = (g \circ f)(x)$

6. <u>Exercises</u>

   (a) Is Function? Section 2.3: 1 - 3.
   (b) Find Domain and Range. Section 2.3: 4 - 7.
   (c) Injective, Surjective, Bijective. Section 2.3: 10 - 23, 74
   (d) Inverses. Section 2.3: 29
   (e) Composition. Section 2.3: 38, 33 - 35