SOLUTION

BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MTH 30 | Exam 1
Kerry Ojakian | SPRING 2026
YOUR NAME (first, then last):

Directions: Write your responses in the provided space. To get full credit you **must** show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. **No** electronic devices are allowed (i.e. no calculators, no phones, no smart watches, etc) - using one during the exam will result in at least a failure on this TEST. Each question is worth 10 points (scaled to total of 100).

1. Consider the following 3 relations. For each one, determine whether it is a function. If it is a function, determine whether it is injective.

   (a) $\{(1,4),(2,5),(3,6),(4,7)\}$

   (b) $\{(0,-1),(2,3),(5,3),(7,8)\}$

   (c) $\{(-2,5),(-2,7),(3,1)\}$

   SOLUTION.
   (1) Each input appears once $\rightarrow$ function. Outputs are distinct $\rightarrow$ injective.
   (2) Each input appears once $\rightarrow$ function. But 2 and 5 both map to 3 $\rightarrow$ not injective.
   (3) Input -2 maps to two different outputs $\rightarrow$ not a function.

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2. What are the absolute extrema of the graph of $y = (x-4)^2 + 3$? (Give the x and y coordinates.)
   SOLUTION.
   This is the graph of $y = x^2$ which has been shifted to the right 4 and up 3, so the minimum at $(0,0)$ has been shifted to $(4,3)$.
   Absolute minimum: $(4,3)$. There is no absolute maximum (the graph increases without bound).

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3. Let $g(x) = 2x^3 - x^4$. Evaluate $g(1)$ and $g(-1)$.
SOLUTION.
$g(1) = 2(1)^3 - (1)^4 = 2 - 1 = 1$.
$g(-1) = 2(-1)^3 - (-1)^4 = -2 - 1 = -3$.

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4. Let $h(t) = 5t$. Evaluate $h(-x)$ and $h(x + 3)$.
SOLUTION.
$h(-x) = 5(-x) = -5x$.
$h(x + 3) = 5(x + 3) = 5x + 15$.

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5. Let $f(x) = \dfrac{x^2 - 16}{(x - 4)(x + 2)}$.

   (a) When is the function undefined?

   (b) What is the domain?

   (c) Evaluate $f(0)$.

   (d) For which x does $f(x) = 0$?

   SOLUTION.
a) Denominator zero when $x = 4$ or $x = -2$, so undefined at $4, -2$.
b) Domain: all real numbers except $x = 4, -2$.
c) $f(0) = \frac{-16}{(-4)(2)} = \frac{-16}{-8} = 2$.
d) $f(x) = 0$ when numerator $x^2 - 16 = 0$, so solve to see $x = \pm 4$. But $x = 4$ undefined $\rightarrow$ only $x = -4$.

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6. Let $f(x) = \dfrac{3}{x - 1}$ and $g(x) = \sqrt{2x - 6}$. Find the domain of:

   (a) $(f + g)$ (b) $(f - g)$ (c) $(f \cdot g)$

   SOLUTION.
f undefined at $x = 1$. g requires $2x - 6 \geq 0 \Rightarrow x \geq 3$.
Intersection $\rightarrow x \geq 3$.
Domain for all three: $[3, \infty)$.

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7. Let $f(x) = x^2 - 9$ and $g(x) = \dfrac{2}{x-2}$. Find the domain of:

   (a) $\dfrac{f}{g}$

   (b) $\dfrac{g}{f}$

   SOLUTION.
   g undefined at $x = 2$. $f = 0$ at $x = \pm 3$.
   (a) $\frac{f}{g}$: denominator $g(x)$ is never zero, but g is undefined when $x = 2$. So domain: all reals except 2.
   (b) $\frac{g}{f}$: denominator $f(x)$ is zero when $x = \pm 3$, and g is undefined when $x = 2$, so domain: all reals except $-3, 2, 3$.

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8. Let $f(x) = 3x + 2$ and $g(x) = 4x - 5$.

   (a) Find $(f \circ g)(2)$.

   (b) Simplify $f(g(x))$.

   (c) Simplify $(g \circ f)(x)$.

   SOLUTION.
   (a) $(f \circ g)(2) = f(g(2)) = f(3) = 11$
   (b) $f(g(x)) = 3(4x - 5) + 2 = 12x - 15 + 2 = 12x - 13$.
   (c) $g(f(x)) = 4(3x + 2) - 5 = 12x + 8 - 5 = 12x + 3$.

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9. Let $h(x) = (x + 2)^2 - 4$.

   (a) Graph the basic function from which $h(x)$ is transformed.

   (b) Graph $h(x)$ using transformations.

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10. Write a formula for the function obtained when the graph of $f(x) = \sqrt{x}$ is shifted down 1 unit and left 4 units.
    SOLUTION.
    Shift left 4 $\rightarrow$ replace x with $x + 4$. Shift down 1 $\rightarrow$ subtract 1.
    New function: $y = \sqrt{x + 4} - 1$.

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11. Consider the function $f(x) = -(x-2)^2$.

   (a) Where is it increasing?

   (b) Where is it decreasing?

   (c) Find the absolute extrema (if it does not have a max or min, state this).

   (d) Find the local extrema (if it does not have a max or min, state this).

   SOLUTION.
   This is $y = x^2$ shifted to the right 2, then reflected across the x-axis. So it has an absolute maximum at $(2,0)$, increasing up to that point, then decreasing. Thus we have th following.
   a) Increasing on interval $(-\infty, 2)$
   b) Decreasing on interval $(2, +\infty)$
   c) Absolute max at $(2,0)$. No absolute min, since it decreases without bound.
   d) Local max also at $(2,0)$. No local min.

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12. Evaluate $f(-3)$, $f(9)$, and $f(11)$ where

   $$f(x) = \begin{cases} 2x & \text{if } x \leq 9 \\ x+7 & \text{if } x > 9 \end{cases}$$

   Also determine whether f is one-to-one.
   SOLUTION.
   $f(-3) = 2(-3) = -6$.
   $f(9) = 2(9) = 18$.
   $f(11) = 11 + 7 = 18$.
   Not one-to-one because, for example, $f(9) = 18 = f(11)$.

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13. Consider the function f given by the following table.

x	-4	-2	0	3	6
$f(x)$	10	8	5	1	-2

(a) Evaluate $f(0)$.

(b) Evaluate $f(-4)$.

(c) For what x does $f(x) = 1$?

(d) Is the function increasing or decreasing?

SOLUTION.
(1) $f(0) = 5$.
(2) $f(-4) = 10$.
(3) $f(x) = 1$ when $x = 3$.
(4) Outputs decrease as x increases $\rightarrow$ decreasing function.

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14. Consider the function graphed below.

(a) Find the intervals where it increases.

(b) Find the intervals where it decreases.

(c) Find all local maxima (both x and y coordinates).

(d) Find all local minima (both x and y coordinates).