SOLUTIONS

BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MTH 30 Exam 3
Kerry Ojakian SPRING 2026
YOUR NAME (first, then last):

Directions: Write your responses in the provided space. To get full credit you **must** show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. **No** electronic devices are allowed (i.e. no calculators, no phones, no smart watches, etc) - using one during the exam will result in at least a failure on this test. Each question is worth 10 points (total scaled to 100).

1. Determine the end behavior of the functions.

   (a) $g(x) = -3x^8$ (b) $h(x) = 4x^3 + 10x^7$

   SOLUTION.

   For (a).
   As $x \to +\infty,\ g(x) \to -\infty$ (since exponent even and coefficient negative)
   As $x \to -\infty,\ g(x) \to -\infty$ (since exponent even and coefficient negative)

   For (b).
   As $x \to +\infty,\ h(x) \to +\infty$ (by test value in leading term: $10(1)^7 = 10 > 0$)
   As $x \to -\infty,\ h(x) \to -\infty$ (by test value in leading term: $10(-1)^7 = -10 < 0$)

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2. Simplify.

(a) $\log_b b^5$ (b) $\log_b \frac{1}{b^3}$

SOLUTION.

Using exponent rules.

a) $\log_b b^5 = 5 \log_b b = 5 \cdot 1 = 5$

b) $\log_b \frac{1}{b^3} = \log_b b^{-3} = (-3) \log_b b = (-3) \cdot 1 = -3$

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3. Evaluate.

(a) $\log_{10}(1)$ (b) $\log_5(25)$ (c) $\log_2(\frac{1}{8})$

SOLUTION.

$\log_{10}(1) = 0$ because $\log_{10}(1) = x$ equivalent to $10^x = 1$, so $x = 0$.
$\log_5(25) = 2$ because $\log_5(25) = x$ equivalent to $5^x = 25$, so $x = 2$.
$\log_2\left(\frac{1}{8}\right) = -3$ because $\log_2(\frac{1}{8}) = x$ equivalent to $2^x = \frac{1}{8}$, so $x = -3$.

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4. (a) Condense $2 \log_5(M) + 5 \log_5(N)$ into a single log.

(b) Use log properties to expand $\log_3\left(\frac{a^3}{b^4}\right)$.

SOLUTION.

(a). $2 \log_5(M) + 5 \log_5(N) = \log_5(M^2) + \log_5(N^5) = \log_5(M^2 N^5)$

(b). $\log_3\left(\frac{a^3}{b^4}\right) = \log_3(a^3) - \log_3(b^4) = 3 \log_3(a) - 4 \log_3(b)$

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5. Solve.
$\log_3(y + 2) + \log_3(y - 2) = 4$

SOLUTION.

$\log_3((y + 2)(y - 2)) = 4$
$\log_3(y^2 - 4) = 4$
$y^2 - 4 = 3^4$
$y^2 - 4 = 81$
$y^2 = 85$
$y = \pm\sqrt{85}$

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6. Graph the following function, drawing its asymptotes as dashed lines (also, give the equation of each asymptote).

$$h(x) = 2 - \frac{2}{(x-3)^3}$$

SOLUTION.

To graph h we can start with the basic function $f(x) = \frac{1}{x^3}$ which has a vertical asymptote of $x = 0$ and a horizontal asymptote of $y = 0$.
To get h, we shift f up 2, so h has horizontal asymptote $y = 2$.
And we shift f to the right 3, so h has a vertical asymptote of $x = 3$.

To graph the function:
1. Plot the horizontal asymptote: $y = 2$ (as a dashed line). 2. Plot the vertical asymptote: $x = 3$ (as a dashed line). 3. Sketch h with the proper shape.

PUT GRAPH! ...

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7. Solve. $3^{x+2} = 81$

SOLUTION.

$3^{x+2} = 81$.
First, express 81 as a power of 3: $81 = 3^4$.
Now, rewrite the equation as $3^{x+2} = 3^4$.
Since the bases are the same, equate the exponents: $x + 2 = 4$.
Therefore, $x = 4 - 2 = 2$.

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8. Let $f(x) = \dfrac{4x^2 + 7}{x^2 - x - 12}$.

   (a) What is the domain?

   (b) What are the vertical asymptotes?

   (c) What is the horizontal asymptote?

   SOLUTION.

   (a) The domain consists of all real numbers for which the denominator is not zero.

   Factor the denominator: $x^2 - x - 12 = (x - 4)(x + 3)$

   So the denominator is zero when $x = 4 \quad \text{or} \quad x = -3$.

   Therefore, the domain is $(-\infty, -3) \cup (-3, 4) \cup (4, \infty)$.

   (b) Vertical asymptotes occur where the denominator is zero (and as a detail, the numerator should be nonzero which is true, since $4x^2 + 7 > 0$).

   For all real numbers, the vertical asymptotes are

   $$x = -3 \quad \text{and} \quad x = 4.$$

   (c) Since the degrees of the numerator and denominator are the same, the horizontal asymptote is the ratio of the leading coefficients:

   $y = \frac{4}{1} = 4$.

   So the horizontal asymptote is $y = 4$.

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9. Solve the equation $x^3 - 4x^2 + 5x - 20 = 0$ given that 2 is a zero.

   SOLUTION.

   2 is NOT a zero! Problem thrown out ...

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10. Consider the linear equation $g(x) = 5x + 3$.

(a) Write the equation for a line parallel to $g(x)$ and passing through the point $(1, -4)$.

(b) Write the equation for a line perpendicular to $g(x)$ and passing through the point $(2, 3)$.

SOLUTION.

The equation of a line parallel to $g(x) = 5x + 3$ has the same slope, 5. Using the point-slope form $y = y_1 + m(x - x_1)$ with point $(1, -4)$:

$$y = -4 + 5(x - 1) \quad \Rightarrow \quad y = 5x - 5 - 4 \quad \Rightarrow \quad y = 5x - 9$$

The equation of a line perpendicular to $g(x) = 5x + 3$ requires the negative reciprocal of the slope, $-\frac{1}{5}$. Using the point-slope form with point $(2, 3)$:

$$y = 3 - \frac{1}{5}(x - 2) \quad \Rightarrow \quad y = -\frac{1}{5}x + \frac{2}{5} + 3 \quad \Rightarrow \quad y = -\frac{1}{5}x + \frac{17}{5}$$

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11. For each function, find its domain and range.

(a) $g(x) = 5 + \sqrt{9x + 4}$

(b) $k(x) = 2 - \ln(3x - 7)$

SOLUTION.

For $g(x) = 5 + \sqrt{9x + 4}$:
Domain: Solve $9x + 4 \geq 0$.
$9x \geq -4$
$x \geq -\frac{4}{9}$
Domain: $\left[-\frac{4}{9}, \infty\right)$

Range: Since $\sqrt{9x + 4} \geq 0$, $g(x) \geq 5$.
Range: $[5, \infty)$

For $k(x) = 2 - \ln(3x - 7)$:
Domain: Solve $3x - 7 > 0$.
$3x > 7$
$x > \frac{7}{3}$
Domain: $\left(\frac{7}{3}, \infty\right)$

Range: Since $\ln(3x - 7)$ ranges over $(-\infty, \infty)$, so does $2 - \ln(3x - 7)$ since this is a shift up and a reflection across the x-axis.
Range: $(-\infty, \infty)$

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12. Find the possible rational zeros of the polynomial $3x^5 + 2x^3 + 5x - 12$ (all fractional answers should be simplified, and do not repeat the same answer twice).

SOLUTION.

$$\frac{\text{Factors of } -12}{\text{Factors of } 3} =$$

$$= \pm\frac{1}{3},\ \pm\frac{2}{3},\ \pm\frac{3}{3},\ \pm\frac{4}{3},\ \pm\frac{6}{3},\ \pm\frac{12}{3},\ \pm\frac{1}{1},\ \pm\frac{2}{1},\ \pm\frac{3}{1},\ \pm\frac{4}{1},\ \pm\frac{6}{1},\ \pm\frac{12}{1}$$

$$= \pm\frac{1}{3},\ \pm\frac{2}{3},\ \pm 1,\ \pm\frac{4}{3},\ \pm 2,\ \pm 4,\ \pm 2,\ \pm 3,\ \pm 6,\ \pm 12$$

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13. Let $g(x) = -3(x-12)^5(x+1)^2(x-7)$. Find its zeroes and the multiplicity of each.

SOLUTION.

12 of multiplicity 5, since the corresponding factor of $(x-12)$ has an exponent of 5.
-1 of multiplicity 2, since $(x+1)$ has an exponent of 2.
7 of multiplicity 1, since $(x-7)$ has an exponent of 1.
