WebHW 10 Fall2025 RationalFunct

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closes 04/13/2026 at 11:00pm EDT

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Problem 1. **(1 point)**

Domains of Rational Functions

Practice

State the domain of the function $f(x) = \dfrac{4x+5}{9x+4}$ in interval notation.

- Type "inf" and "-inf" for ∞ and $-\infty$, respectively.
- Type the capital letter "U" to represent the union symbol $\cup$.

The domain of $f(x)$ is ______

Correct Answers:

- $\left(-\infty, \frac{-4}{9}\right) \cup \left(\frac{-4}{9}, \infty\right)$

Problem 2. **(1 point)**

State the domain of the function $f(x) = \dfrac{5x+6}{9x^2+18x+58}$ in interval notation.

- Type "inf" and "-inf" for ∞ and $-\infty$, respectively.
- Type the capital letter "U" to represent the union symbol $\cup$.

The domain of $f(x)$ is ______

Correct Answers:

- $(-\infty, \infty)$

Problem 3. **(1 point)**

Intercepts of Rational Functions

Practice

For the function $f(x) = \dfrac{9x-2}{x^2+5x+6}$,

a. What is the y-intercept? ____

b. What are the x-intercepts? ______

- Enter your answers as points: (x, y)
- If you have more than one x-intercept, use commas to list the points.

Correct Answers:

- $\left(0, \frac{-1}{3}\right)$
- $\left(\frac{2}{9}, 0\right)$

Problem 4. **(1 point)**

Domains of Rational Functions

Practice

State the domain of the function $f(x) = \dfrac{8x-7}{3x^3-16x^2+x+60}$ in interval notation.

- Type "inf" and "-inf" for ∞ and $-\infty$, respectively.
- Type the capital letter "U" to represent the union symbol $\cup$.

The domain of $f(x)$ is ______

Correct Answers:

- $\left(-\infty, \frac{-5}{3}\right) \cup \left(\frac{-5}{3}, 3\right) \cup (3, 4) \cup (4, \infty)$

Problem 5. **(1 point)**

Intercepts of Rational Functions

Consider $f(x) = \dfrac{x^2 - 1}{2x^2 - x - 3}$.

To find the x-intercept(s) for $f(x)$, we must find the x-value(s) that cause $f(x) = 0$.
In other words, $x^2 - 1 = 0$ or, when factored, $(x-1)(x+1) = 0$, which tells us that the x-values we seek are $x = -1$ and $x = 1$.

When we evaluate $f(1)$ we get $f(1) = \dfrac{1^2 - 1}{2(1)^2 - 1 - 3} = \dfrac{0}{-2} = 0$ (as we expected, $f(1) = 0$).

But, if we evaluate $f(-1)$ we get $f(-1) = \dfrac{(-1)^2 - 1}{2(-1)^2 - (-1) - 3} = \dfrac{0}{0}$ which is undefined!

What is happening here?

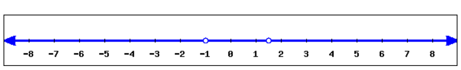
We need to look further into the x-intercept discussion in relation to the domain.

So, let's find the domain of $f(x)$:

Recall that to find the domain of a rational function, we exclude all the values of x that would make the denominator 0.

For $f(x)$, we need to make sure that our denominator, $2x^2 - x - 3$, is *not* zero. Factoring, we see that $(x+1)(2x-3) \neq 0$, and so $x \neq -1$ and $x \neq \dfrac{3}{2}$.

The domain of this function is $(-\infty, -1) \cup (-1, 3/2) \cup (3/2, \infty)$ and the graph of the domain is:



Since $x = 1$ is in the domain, we have no conflict and we can say that $(1, 0)$ is an x-intercept for $f(x)$.

On the other hand, $x = -1$ is *not* in the domain of $f(x)$:
- This explains why we did not get an output-value for $x = -1$;
- and without a y-value we cannot have a point;
- And without a point for $x = -1$, we cannot have an intercept there.

So $f(x)$ has only one x-intercept, the point $(1, 0)$.

Note:
- You must **always** consider the domain of $f(x)$, even when you're not explicitly asked to find it.
- You may want to start by finding the domain at the beginning of each problem.
- If you don't find the domain at the start of each problem, you **certainly** must double-check that your x-values produce the expected output-value.

Practice

For the function $f(x) = \dfrac{x^2 + 12x + 32}{x^2 + 13x + 40}$,

a. What is the y-intercept? ____

b. What are the x-intercept(s)? _______

- Enter your answers as points: (x, y)
- If you have more than one x-intercept, use commas to list the points.

Correct Answers:
- $\left(0, \dfrac{4}{5}\right)$
- $(-4, 0)$

Problem 6. **(1 point)**

Vertical Asymptotes

Because rational functions involve division, we must be careful in avoiding the input of any x-values that would cause division by 0.

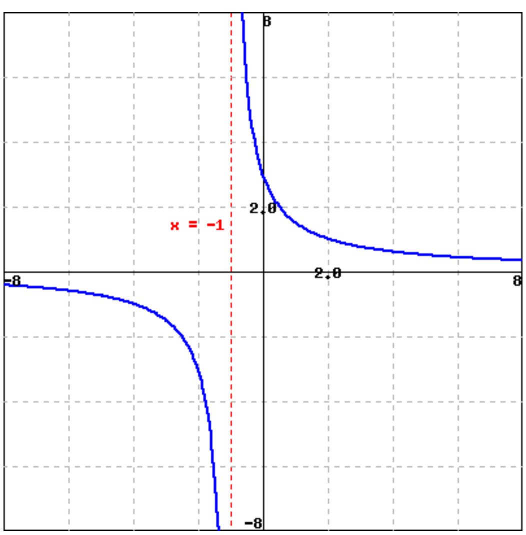
For example, the function $f(x) = \dfrac{3}{x+1}$ requires us to divide by a value which is one larger than the input value. In other words, we divide by $x+1$.

This will be problematic if we were to input a value of -1 for x, because $f(-1) = \dfrac{3}{-1+1} = \dfrac{3}{0}$, which is undefined.

Since the input-value of $x = -1$ does not have a corresponding output-value, we must exclude $x = -1$ from the domain.

In interval notation, the domain of $f(x)$ is $(-\infty, -1) \cup (-1, \infty)$.

Let us now consider the graph of $f(x) = \dfrac{3}{x+1}$, and focus on the behavior of our graph for inputs close to $x = -1$:



Because our function must avoid $x = -1$, we have marked the corresponding vertical line with a dashed red line. You should see that as the input values get close to -1, the function curves to approach this vertical line, $x = -1$.

We describe this behavior as "*asymptotic*", and we refer to the line $x = -1$ as a "*vertical asymptote*" of $f(x)$.

This behavior emerges because we are attempting to divide 3 by values that are incredibly close to zero.

- When x is very close to -1, the denominator, $x+1$, will be very close to zero.
- For instance, if $x = -0.99$, our denominator would be $(-0.99 + 1)$ or 0.01.

When 3 is divided by a value close to zero, say 0.01, it will become much larger.

- $3 \div 0.01 = 300$
- Think about what would happen if we were even closer to zero.
- What if we divided $3 \div 0.00001$?

On the other side of $x = -1$, our denominator will be negative-valued, but still close to zero.

- What happens if we divide $3 \div -0.00001$?

Practice

- If you have more than one vertical asymptote, separate them using commas.

a. $f(x) = \dfrac{3}{x-3}$

What is (are) the vertical asymptote(s) of $f(x)$? ________

b. $g(x) = -\dfrac{3}{x-2}$

What is (are) the vertical asymptote(s) of $g(x)$? ________

c. $h(x) = \dfrac{2}{x^2-9}$

What is (are) the vertical asymptote(s) of $h(x)$? ________

Correct Answers:

- $x = 3$
- $x = 2$
- $x = 3, x = -3$

Problem 7. **(1 point)**
Consider the function

$$f(x) = \frac{-2}{4x+6}$$

Find the vertical asymptote(s). If there is more than one vertical asymptote give a list of the x-values separated by commas.

$x =$ ________________

If this function has a horizontal asymptote, give its y-value. If there is no horizontal asymptote, type in *none* .

____

Find the x-intercept(s). If there is more than one x-intercept give a list of the x-values separated by commas. If there is no x-intercept type in *none* .

$x =$ ________________

Find the y-intercept

$y =$ ________________

Correct Answers:

- -1.5
- 0
- None
- -0.333333333333333

Problem 8. **(1 point)**
Let $f(x) = \dfrac{2}{5x^2 - 41x + 42}$. What is (are) the vertical asymptote(s) of $f(x)$?

- If you have more than one vertical asymptote, separate them using commas.

____

Correct Answers:

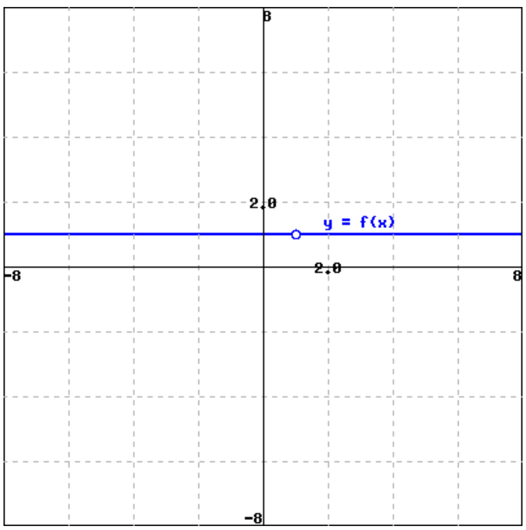
- $x = \left(\frac{6}{5}\right), x = 7$

Problem 9. **(1 point)**

Vertical Asymptote vs Hole

For the function $f(x) = \dfrac{x-1}{x-1}$, we'd like to say that this simplifies to being the same as $f(x) = 1$; and we'd be *mostly* correct, except for the fact that $f(x) = \dfrac{x-1}{x-1}$ is *undefined* when $x = 1$. However, for **every other** x-value (besides $x = 1$), $f(x) = 1$.

This gives us a graph that looks like:



What we should notice here is that *even though* we have a "domain issue" at $x = 1$, we **don't** have a vertical asymptote.

This can be recognized in *any* function whose output looks like "$\frac{0}{0}$". Now we must realize that $\frac{0}{0}$ is not a real number, and therefore does not constitute a valid output from our function. However, the appearance of $\frac{0}{0}$ does tell us *something* about our rational function.

When faced with a $\frac{0}{0}$ situation, what we should realize is that this situation indicates that *both* our numerator **and** our denominator have a common **root**. Furthermore, this common **root** indicates that our numerator and denominator share a **factor** (because every root gives us a factor and every factor gives us a root). And the presence of a common factor in our numerator and denominator tells us that we can *reduce* our rational function (everywhere except at the root, because the $\frac{0}{0}$ output is still undefined).

Practice

a. $y = \dfrac{x-4}{x-4}$ has a domain issue at $x = 4$. Will there be a vertical asymptote at $x = 4$? [?/Yes/No]

b. $y = \dfrac{x^2+16}{x-4}$ has a domain issue at $x = 4$. Will there be a vertical asymptote at $x = 4$? [?/Yes/No]

c. $y = \dfrac{x+4}{x-4}$ has a domain issue at $x = 4$. Will there be a vertical asymptote at $x = 4$? [?/Yes/No]

d. $y = \dfrac{x^2-16}{x-4}$ has a domain issue at $x = 4$. Will there be a vertical asymptote at $x = 4$? [?/Yes/No]

Correct Answers:

- No
- Yes
- Yes
- No

Problem 10. **(1 point)**

Horizontal Asymptotes

Recall that a rational function admits a horizontal asymptote under two situations:

- Degree of the leading terms of both numerator and denominator are the same. In this case, the equation of the asymptote is given by the ratio of the leading coefficients.

- Degree of the leading terms of the numerator is less than that of the denominator. In this case, the equation of the asymptote is always $y = 0$.

Practice

What is the horizontal asymptote for $y = \dfrac{2x}{x-1}$? ____

What is the horizontal asymptote for $y = \dfrac{-5x}{x-1}$? ____

What is the horizontal asymptote for $y = \dfrac{-9x}{2x-1}$? ____

Correct Answers:

- $y = 2$
- $y = -5$
- $y = -\dfrac{9}{2}$

Problem 11. **(1 point)**

Rational Functions and Asymptotes

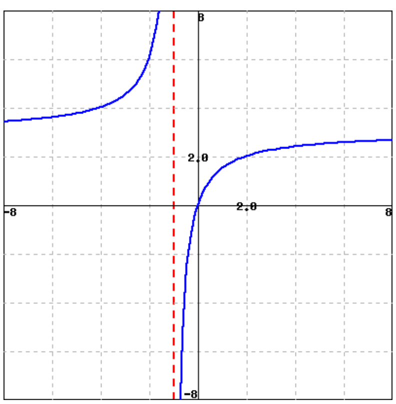
Features that are particular to graphs of rational functions (as opposed to those of polynomials) are horizontal and vertical asymptotes and holes.

We can say a lot about the graph of a rational function by analyzing its equation in regards to the three features above.

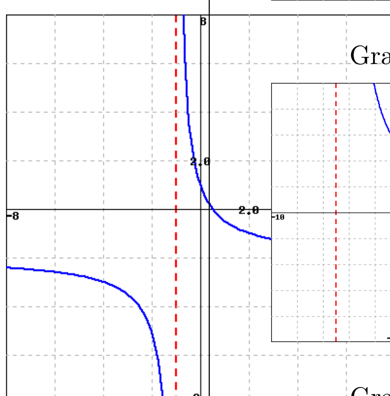
For example, consider $f(x) = \dfrac{3x}{x+1}$. The domain is all values of x except $x = -1$ since $x + 1$ cannot be 0.

Moreover, since $(x + 1)$ is not a factor of the numerator, $x = -1$ is a vertical asymptote, and we have no holes in the graph of $f(x)$.

The following graphs reflect a vertical asymptote at $x = -1$:



Graph 1



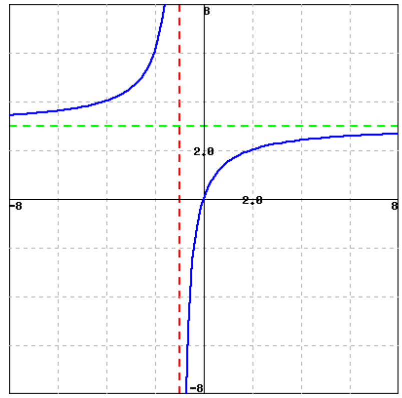
Graph 2

Becasue $f(x)$ has no holes, Graph 3 is not a suitable image for the graph of $f(x)$.

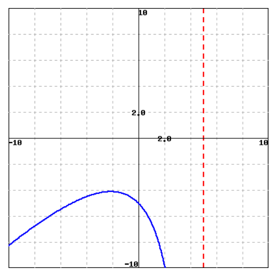
However, $f(x)$ has a horizontal asymptote at $y = \dfrac{3}{1} = 3$ is . So the graph of $f(x)$ should reflect:

- Horizontal asymptote $y = 3$
- Vertical asymptote $x = -1$
- No holes

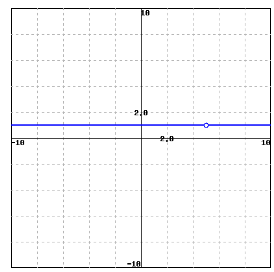
This leaves us with Graph 1 as the best option for the graph of $f(x)$.



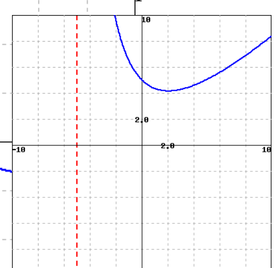
Practice



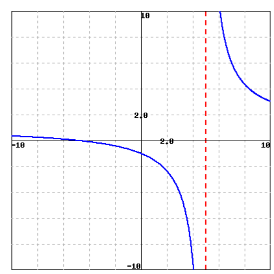
Graph A



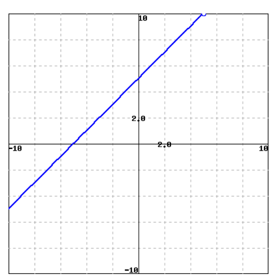
Graph B



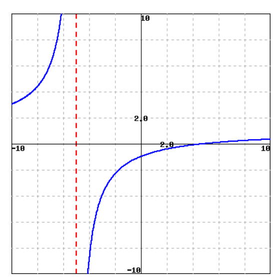
Graph C



Graph D



Graph E



Graph F

1. $y = \dfrac{x-5}{x+5}$ is shown in graph [?/A/B/C/D/E/F]

2. $y = \dfrac{x^2+25}{x-5}$ is shown in graph [?/A/B/C/D/E/F]

3. $y = \dfrac{x^2-25}{x-5}$ is shown in graph [?/A/B/C/D/E/F]

4. $y = \dfrac{x^2+25}{x+5}$ is shown in graph [?/A/B/C/D/E/F]

Correct Answers:

- F
- A
- E
- C

Problem 12. **(1 point)**

Vertical Asymptotes and Holes

Let's consider the rational function $f(x) = \dfrac{x}{(x+1)(x+2)}$.

Recall that:

- the denominator of a rational function not only gives us the domain, but it produces the possible vertical asymptotes and possible holes.
- a hole on the graph of a rational function comes from a common factor between the numerator and denominator.

Is $x = -1$ a vertical asymptote or a hole for $f(x)$? What about $x = -2$?

Now, let's consider $g(x) = \dfrac{x+1}{(x+1)(x+2)}$.

Is $x = -1$ a vertical asymptote or a hole for $g(x)$? What about $x = -2$?

Let's analyze:

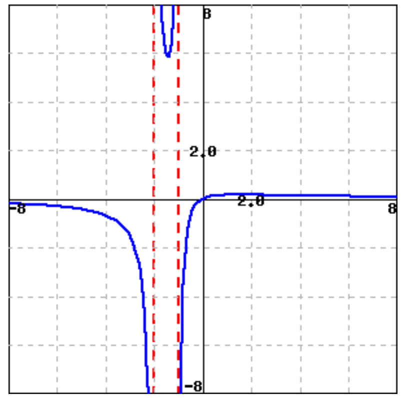
The domain of both functions is $(-\infty,-2) \cup (-2,-1) \cup (-1,\infty)$.

$f(x)$ will have two vertical asymptotes $x = -2$ and $x = -1$.

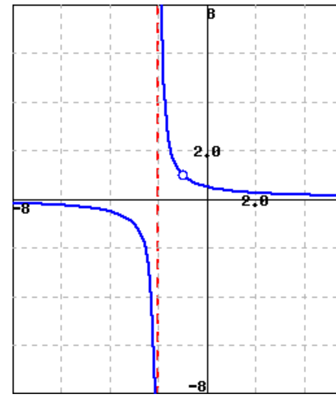
On the other hand $g(x)$ will have a vertical asymptotes at $x = -2$ and a hole at $x = -1$. (Notice how (x+1) is a common factor between the numerator and the denominator).

By the way, both functions have a horizontal asymptote at $y = 0$ because the degree of the numerator is smaller than that of the denominator in both.

We verify all the above with the graphs:



f(x)



g(x)

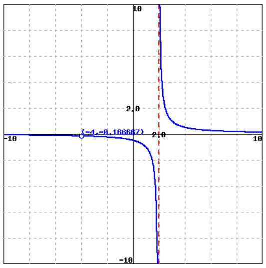
We can also work backwards and deduce from the graph the factors present in the numerator, denominator or both for a rational function. For instance:

- a hole gives us the same factor in both numerator and denominator.
- a vertical asymptote gives us a factor in the denominator (that is not in the numerator).
- a root gives us a factor in the numerator (that is not in the denominator).

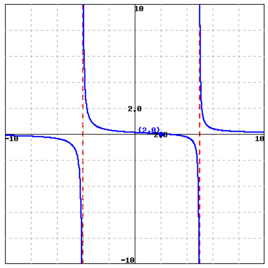
Try to look at the graphs of $f(x)$ and $g(x)$ above and see if you can work out the pieces of the equation of each function.

Can you see how we can also read off the domain of $f(x)$ and $g(x)$ from its corresponding graph?

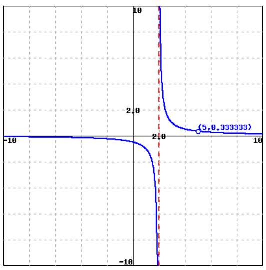
Practice



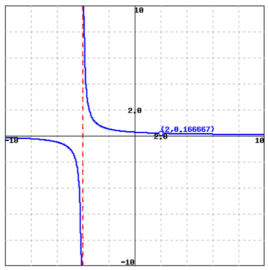
Graph A



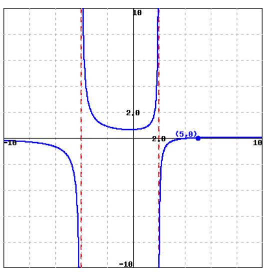
Graph B



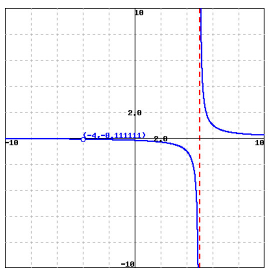
Graph C



Graph D



Graph E



Graph F

1. $y = \dfrac{x+4}{(x+4)(x-5)}$ is shown in graph [?/A/B/C/D/E/F]
2. $y = \dfrac{x-5}{(x-2)(x+4)}$ is shown in graph [?/A/B/C/D/E/F]
3. $y = \dfrac{x+4}{(x-2)(x+4)}$ is shown in graph [?/A/B/C/D/E/F]
4. $y = \dfrac{x-5}{(x-2)(x-5)}$ is shown in graph [?/A/B/C/D/E/F]

Correct Answers:

- F
- E
- A
- C

Problem 13. (1 point)

$$y = \frac{7(x-1)^2(x-7)}{(x-7)(x-4)(x+2)}$$

The domain of this function is: __________

This graph has horizontal asymptote(s) at: __________

This graph has vertical asymptote(s) at: __________

This function has root(s) at: __________

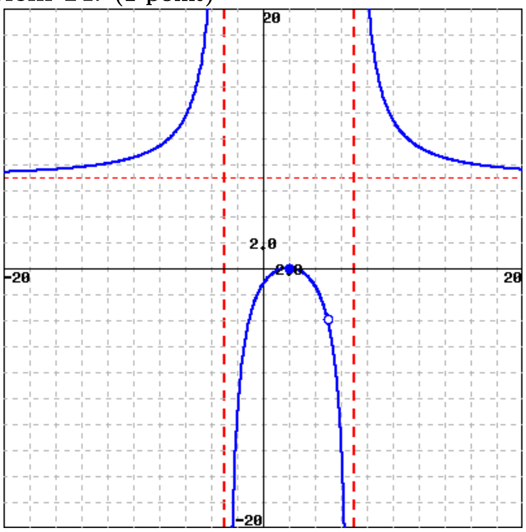
This function has hole(s) at: __________

- If this function does not have one of the requested features, respond with “none”

Correct Answers:

- $(-\infty,-2)\cup(-2,4)\cup(4,7)\cup(7,\infty)$
- $y=7$
- $x=4, x=-2$
- $(1,0)$
- $(7,9.33333)$

Problem 14. (1 point)



The domain of this function is: __________

This graph has horizontal asymptote(s) at: __________

This graph has vertical asymptote(s) at: __________

This function has a root at: __________

Give an equation for the graph of this rational function:
$y =$__________

Correct Answers:

- $(-\infty,-3)\cup(-3,5)\cup(5,7)\cup(7,\infty)$
- $y=7$
- $x=7, x=-3$
- $(2,0)$
- $\frac{7(x-2)^2\,(x-5)}{(x-5)\,(x-7)\,(x+3)}$

Problem 15. (1 point)

Consider the function

$$f(x)=\frac{x^2-4x-21}{x^2+5x}$$

Find the vertical asymptote(s). If there is more than one vertical asymptote give a list of the x -values separated by commas.

$x =$ __________________

If this function has a horizontal asymptote, give its y-value. If there is no horizontal asymptote, type in *None* .

____

Find the x-intercept(s). If there is more than one x-intercept give a list of the x-values separated by commas.

$x =$ __________________

Find the y -intercept. If there is no y -intercept type in *None* .

$y =$ __________________

Correct Answers:

- $-5, 0$
- 1
- $7, -3$
- None