MTH 30 LECTURE NOTES (Ojakian)

Topic 18: Exp and Log Applications

OUTLINE

(References: 4.7, 4.8)

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1. Exponential Growth and Decay Models
   (a) Recall:
      i. $y = A_0 \cdot e^{kt}$ (i.e. Use base e)
      ii. A_0 is the starting value and k determines the rate of growth.
      iii. Constant k is positive for growth, and k is negative for decay.
   (b) General approach:
      i. Write down the general equation: $y = A_0 \cdot e^{kt}$
      ii. Based on the problem, fill in the given values (A_0 or k or t or $\frac{y}{A_0}$)
      iii. Solve for the missing values (A_0 or k or ...)
      iv. Now answer any questions using your filled out equation.
2. Exponential Growth
   (a) Doubling time: Amount of time for quantity to double in size.
   (b) Example: Suppose there is a population of 10 dogs in The Bronx and the current year is 2025. Their doubling time is 2 years. What is their population in 2027? What is their population in 2031?
   (c) Example. Section 4.7: Example 1 - Do three ways: From scratch, just doubling, and using equation, and using doubling time
   (d) Calculate. Doubling time = $\frac{\ln 2}{k}$
   (e) Exercises.
      i. Section 4.7: 37 (find model from doubling time)
3. Exponential Decay
   (a) Half-life: Amount of time for quantity to half in size.
   (b) Example: Suppose there is a substance of in a Manhattan lab that weighs 64 pounds, and the current year is 2025. The substances half-life is 5 years. What is its weight in 2030? What is its weight in 2040?
   (c) Example: Suppose there is a material in a Brooklyn brewery that weighs 100 pounds. The material has a half-life of 10 years. How many half-lives will have passed before the material weighs 25 pounds? How many half-lives will have passed before the material weighs less than a pound?

(d) Example: Suppose an exponential decay model has $k = -3$ and time is measured in hours. After some time, only 10% of the original amount is left. How much time passed.

(e) Application: Radiocarbon Dating. See Section 4.7 page 503.

  i. Use Carbon-14, which has a half-life: 5730 years.
  ii. When a creature is alive, it maintains its Carbon-14 level.
  iii. Once the creature dies, the Carbon-14 decays exponentially
  iv. So we can approximate when it died by finding out how much Carbon-14 its dead remains have!

(f) Calculate. Half-life $= -\dfrac{\ln 2}{k}$

(g) Exercises.

  i. Section 4.7: 28 - 30 (drug problem) and 31, 32 similar
  ii. Section 4.7: 33 (Find model and half-life)
  iii. Section 4.7: 34 (Find annual decay rate) and 35 similar
  iv. Section 4.7: 36 (Find age from ratio of remaining stuff)

4. Logistic Growth

Growth which begins looking exponential, but then plateaus.

Example: Tap the table once in 5 seconds. Then keep doubling, so it looks exponential, but levels out ...

$L(t) = \dfrac{C}{1 + ae^{-bt}}$ (C is the "Carrying Capacity")

(a) Exercises. Section 4.7: 7, 8, 9, 10 (all one model); similar: 50, 51, 52.

(b) Exercises. Section 4.7: 17, 18, 20, 21, 22

5. Fit Data

Fit data for Exponential Model using Google Sheets.

Do exactly as we did for lines, EXCEPT: After choosing "Trendline", toggle to "exponential"

Exercises.

(a) Section 4.8: 26 - 30 (on 30, estimate the answer by repeatedly guessing x values)

(b) Section 4.8: 31 - 35 (like the last problem, with 35 - repeatedly guess x values)