MTH 30 LECTURE NOTES (Ojakian)

Topic 14: Exponential Functions

OUTLINE
(References: 4.1, 4.2)

1. Exponential Functions
2. Graphing

1. Exponential Functions

Motivation A: *If an island starts out with 2 people and the population doubles every year, how long will it take for the population to exceed 1 million people?*

Motivation B: *If we cut a square piece of paper in half 20 times, what will the area of the final piece be compared to the original piece be?*

Note: There are various uses of the term "exponential function". The following is a slight variant from the book.

Definition 1. *Let b be a real number constant such that $b \neq 1$ and $b > 0$. We define the function which is called the* ***exponential function with initial value*** *A_0,* ***base*** *b* ***and growth rate*** *k by:*

$$f(t) = A_0 \cdot b^{kt}$$

PROBLEM 1. *Evaluate the following exponential functions and state what the base is.*

(a) $f(4)$, *where* $f(t) = 10^t$.
(b) $g(2)$ *where* $g(x) = (1/3)^x$.

PROBLEM 2. *For each of the following functions, is it an exponential function? If yes, then also find its base.*

(a) Is $f(x) = 7 \cdot 2^x$.
(b) Is $f(x) = x^3$.
(c) Is $f(x) = (1/2)^x$.
(d) Is $f(x) = 0^x$.

Question: *Why the particular constraints on the base of an exponential function?*

PROBLEM 3. *Answer the paper cutting question above.*

PROBLEM 4. *If an island starts out with 2 people and the population doubles every 1 year, how long will it take for the population to exceed 1 million people?*

Exercises. Section 4.1: 9, 10, 11, 12 (base versus initial value)

2. Graphing Exponential Functions

   (a) Shape and asymptote

   (b) Domain and Range

   (c) Graph Transformations

   (d) Exercises: Do some basic functions, like $f(x) = 2 \cdot 3^x$

3. Speed Comparison - Exponential Functions versus Other types

   Speed comparison (do in Desmos)

   (a) Versus Linear

   (b) Versus Polynomial

4. Growth, and decay

   Assuming k is positive.

   (a) Exponential growth
   $A > 0$ and $B > 1$ (like population growth)

   (b) Exponential decay
   $A > 0$ and $B < 1$ (like cutting paper in half)

   (c) Exercises. Section 4.1: 14 - 17 (identifying growth versus decay)

   (d) Exercises. Section 4.1: 18, 19 (finding model from two function values)

   (e) Exercises. Section 4.1: 62 (fuller problems)

5. Percent Change

   (a) Example: Suppose the price of your favorite food (let's choose this!) increased by 50%. What is the new price?

   (b) Example: Suppose the number of people in your neighborhood (let's choose this!) decreased by 50%. What is the new population?

   (c) In general: An increase at rate r (as decimal!) means: Go from A to $A + rA$

   (d) In general: A decrease at rate r means: Go from A to $A - rA$

   (e) Examples...

6. Application: Compound Interest

   (a) Terminology:

      i. Principle: "P" Amount of initial money
      ii. (Yearly) Interest rate: "r" (as a decimal number, not percentage)

   (b) Investment (with NO compounding)

      i. Formula: $A = P + Prt$
      ii. Example: Consider principle (you choose!) and a rate (you choose). After some number of years, how much do you have?

   (c) Investment (WITH compounding)

      i. Key formula for amount after t years (work it out!):
      $A(t) = P(1 + r)^t$
      ii. Example: Work out your gain from above example, with the same parameters, but now with yearly compounding.
      iii. Exercise. Section 4.1: 31.
         A. First, what if you just get yearly interest without compounding?
         B. Second, do 31 (but do "annual compounding").
      iv. Exercises. Section 4.1: 63, 64

7. Exponential Change is Percent Change

   Actually all exponential change like above example.

   (a) Linear: Constant "addition" with each horizontal step.

      i. If associated slope is m, then each size 1 step adds m.

   (b) Exponential: Constant "percent change" with each horizontal size 1 step.

      i. Consider any exponential growth function $f(x) = A_0 \cdot b^{kt}$.
      ii. For a rate $r = b^k - 1$, we get an equivalent view of f as follows:

$$f(x) = A_0(1 + r)^t$$

   (c) Example: See constant change in $f(x) = 2x$ versus $g(x) = 2^x$.

   (d) Exercises. Section 4.1: 4, 6.

   (e) Exercise. Section 4.1: 61 (percent change)