MTH 30 LECTURE NOTES (Ojakian)

Topic 9: Quadratic Equations and Parabolas

OUTLINE

References: 3.2

1. Parabolas
2. Applications of Parabolas

---

1. Intercepts

   (a)

   PROBLEM 1. *Find the intercepts of the following graphs (indicating which are $x-$intercepts and which are $y-$intercepts)*

   i. $y = x^2 - 9$

   ii. $y = (x+3)(2x-7)$

   iii. $y = x^2 - 2x - 3$

   (b) To find y-intercepts: Set $x = 0$.

   (c) To find x-intercepts: Set $y = 0$.

2. Parabolas

   Definition 1. *A **parabola** is a graph of an equation in one of the following forms*

   $$y = ax^2 + bx + c \quad \text{“General Form” (where } a, b, \text{ and } c \text{ are constants)}$$

   OR

   $$y = a(x-h)^2 + k \quad \text{“Standard Form“ or “Vertex Form” (where } h \text{ and } k \text{ are constants)}$$

   Terminology

   - Open up or down?
   - Axis of symmetry
   - Vertex
   - Symmetric points (about the axis of symmetry)
   - Max and Min

3. Parabolas: by formula

Theorem 1. *For a parabola in general form, its axis of symmetry is given by the equation:* $x = \dfrac{-b}{2a}$

Theorem 2. *For a parabola in vertex form, its vertex is at* (h, k).

PROBLEM 2. *For each equation do the following: 1) Find axis of symmetry, 2) Find the vertex, 3) Find its maximum and minimum values (if they exist), 4) Find its intercepts, 5) Find a pair of points symmetric about the axis of symmetry, and 6) Graph it.*

(a) $y + x^2 = x$

(b) $8x^2 - 2x = y + 1$

4. Applications

(a) Minimize Costs.

PROBLEM 3. *Your factory produces lemon-scented widgets. You know that each unit is cheaper, the more you produce. But you also know that costs will eventually go up if you make too many widgets, due to the costs of storage of the overstock. The guy in accounting says that your cost (C) for producing x thousands of units a day can be approximated by the formula* $C = 2.5x^2 - 30x + 25000$. *Find the daily production level that will minimize your costs (modified from Purple Math).*

(b) Question: Suppose you have some fixed amount of fencing and want to use it to fence in a rectangle. What kind of rectangle maximizes the enclosed area?

PROBLEM 4. *Suppose you have 40 meters of fencing. You want to use the fencing to enclose a rectangular plot of land. You want to enclose as much land as possible. What should you choose as the dimensions of the plot?*

(c) Physics

PROBLEM 5. *Suppose a ball is thrown up into the air by someone standing on the ground. Suppose h is the height of the ball at time t, and the equation relating h and t is:*

$$h = -16t^2 + 96t$$

Find the maximum height of the ball. Determine when the ball reaches this maximum height. Determine when the ball hits the ground (modified from textbook).