

MTH 30 LECTURE NOTES (Ojakian)

Topic 2: Domain and Range

OUTLINE

(References: 1.2)

1. Domain and Range
 2. Interval notation and set builder notation
 3. Piece-wise Functions
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1. Motivating Example

What is the domain and range of $f(x) = \frac{\sqrt{x}}{x-3}$

2. Reminders on Equations

(a)

Definition 1. Given an equation, a **solution** is a single number that makes it true. The **solution set** is the set of numbers that make it true. To **solve** an equation means to find the solution set.

PROBLEM 1. For each equation, find its solution set. Then determine which are “identities” and which are not

i. $x + 5 = 15$

ii. $x + 2x = 3x$

iii. $y + y = y$

iv. $x = x + 1$

v. $x^2 = 25$

(b) Tools for Solving:

i. Simplifying

ii. Doing the same thing to both sides

(c) Examples of linear equations

PROBLEM 2. Solve each equation, writing a justification for each new line in your solution.

i. $13x + 7 - 10x = -2 - 3$

ii. $4x - 6 = 7x - 3$

iii. $2(3 - 2x) = 2 - (3x - 4)$

3. Meaning of inequality signs

PROBLEM 3. For each statement, is it true or false?

(a) $5 < 6.3$

(b) $0 > \frac{91}{13}$

(c) $-43 > -100$

(d) $5 = 5.03$

(e) $3 \leq 7$

(f) $3 < 3$

(g) $3 \leq 3$

(h) $-\frac{3}{4} \geq -10$

4. Solution set

Definition 2. Given an inequality (with a variable), a **solution** is a single number that makes it true. The **solution set** is the set of numbers that make it true. To **solve** an inequality means to find the solution set.

PROBLEM 4. For each inequality, find three different numbers in its solution set. Then solve it. Then graph the solution on the number line.

(a) $x > -5$

(b) $x \geq -5$

(c) $u \leq 2u$

5. Solving Inequalities

(a) Principle: Use all the usual rules (for equations), **except for the exceptions ...**

PROBLEM 5. For each inequality graph its solution set.

i. $x \geq 5$

ii. $2x \geq 10$

iii. $-2x \geq -10$

iv. $-2x \leq -10$

(b) Principle for solving inequalities: Use all the usual rules (for equations), **except** that when you multiply or divide by a negative number **reverse the inequality**.

(c)

PROBLEM 6. Solve each inequality

i. $\frac{x}{-3} > 6$

ii. $2(x - 7) < -6$

iii. $8y + 3(y - 12) > 7y - 28$

6. Inequality Notation and Interval Notation

- (a) Endpoints that are numbers
- (b) Endpoints that are infinities
- (c) Unions

7. Set Builder Notation

- (a) Examples

8. Using Algebraic Definition versus the Graph

- (a) Examples
- (b) Algebra problems. Section 1.2: 6, 14, 8 (and if 1 over this)
- (c) Graph problems. Section 1.2: 28, 29

9. Piece-wise Defined Functions

- (a) Examples
- (b) Problems. Section 1.2: 39

10. More Problems

Section 1.2: 57