SOLUTION

BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MTH 30 Exam 3
Kerry Ojakian FALL 2025
YOUR NAME (first, then last):

Directions: Write your responses in the provided space. To get full credit you **must** show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. **No** electronic devices are allowed (i.e. no calculators, no phones, no smart watches, etc) - using one during the exam will result in at least a failure on this test. Each question is worth 10 points (total scaled to 100).

1. Evaluate each expression or state why it is undefined.

   (a) $\log_{15}(1)$ (b) $\log_{15}(-1)$ (c) $\log_{19}(\frac{1}{19})$

   SOLUTION:

   a) $\log_{15}(1) = 0$ (since $15^0 = 1$)

   b) $\log_{15}(-1)$ is undefined since the domain of log is positive reals, so it is not defined at -1.

   c) $\log_{19}(\frac{1}{19}) = x$ converted to exponential equation: $19^x = \frac{1}{19}$, so value is: -1.

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2. (a) Condense $3\log(a)+\log(c)-\log(a^2)$ into a single log and simplify. (b) Use log properties to expand $\log_4(\frac{x}{yw})$.

   SOLUTION:

   (a) $= \log(a^3) + \log(c) - \log(a^2) = \log(a^3 \cdot c) - \log(a^2) = \log(\frac{a^3 \cdot c}{a^2}) = \log(a \cdot c)$

   (b) $= \log_4(x) - \log_4(yw) = \log_4(x) - (\log_4(y) + \log_4(w)) = \log_4(x) - \log_4(y) - \log_4(w)$

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3. Solve. $3^{5x-4} = 9$
SOLUTION: $3^{5x-4} = 9$. One approach: Write both sides as powers of 2 and proceed as usual.
An alternative way follows: take $\log_3$ of both sides, $\log_3 3^{5x-4} = \log_3 9$, and simplify: $5x-4 = 2$. Therefore $5x = 6$, so $x = \frac{6}{5}$.

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4. Solve. $\log_2(x-1) + \log_2(x+1) = 3$
SOLUTION:

$$\begin{aligned}
\log_2(x-1) + \log_2(x+1) &= 3 \\
\log_2((x-1)(x+1)) &= 3 \\
(x-1)(x+1) &= 2^3 \\
x^2 - 1 &= 8 \\
x^2 &= 9 \\
x &= 3, -3
\end{aligned}$$

Check $x = 3$: $\log_2((3)-1) + \log_2((3)+1) = \log_2(2) + \log_2(4) = 1 + 2 = 3$.
Check $x = -3$: $\log_2((-3)-1) + \log_2((-3)+1)$ undefined since domain is positive reals.
Answer: 3.

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5. Let $f(x) = \dfrac{x^2+7}{3x^2-2x-1}$.

   (a) What is the domain?

   (b) What are the vertical asymptotes?

   (c) What is the horizontal asymptote?

   SOLUTION:
   Factor bottom: $3x^2 - 2x - 1 = (3x+1)(x-1)$, so it is zero when $x = -\frac{1}{3}, 1$.

   (a) Domain: All reals except $-\frac{1}{3}, 1$

   (b) Vertical asymptotes: $x = -\frac{1}{3}$ and $x = 1$

   (c) Horizontal asymptote: $y = \frac{1}{3}$ (since the ratio of the leading terms is $\frac{x^2}{3x^2} = \frac{1}{3}$)

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6. Find the possible rational zeros of the polynomial $4x^5 + 6x^2 + 4x - 10$ (all fractional answers should be simplified, and do not repeat the same answer twice).
SOLUTION:

$$\frac{\text{Factors of } -10}{\text{Factors of } 4} = \pm 1, \quad \pm\frac{1}{2}, \quad \pm\frac{1}{4}, \quad \pm 2, \quad \pm 5, \quad \pm\frac{5}{2}, \quad \pm\frac{5}{4}, \quad \pm 10$$

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7. Find the quotient and remainder that result from the following division: $\dfrac{4x^3 + x^2 - 10x + 20}{x+2}$

$$
\begin{array}{r|l}
 & 4x^2 - 7x + 4 \\
\hline
x+2 & 4x^3 + x^2 - 10x + 20 \\
 & \underline{4x^3 + 8x^2} \\
 & -7x^2 - 10x \\
 & \underline{-7x^2 - 14x} \\
 & 4x + 20 \\
 & \underline{4x + 8} \\
 & 12
\end{array}
$$

$\Rightarrow$ Quotient: $4x^2 - 7x + 4$

Remainder: 12

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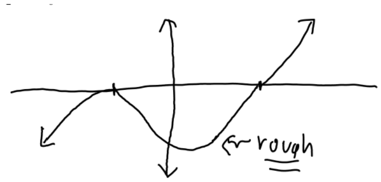
8. Graph $f(x) = 3(x+2)^2(x-3)^3$ roughly: show the x-intercepts and the end behavior. Also find its zeroes and the multiplicity of each.

Zeroes
-2 (mult 2)
3 (mult 3)
Leading term: $3x^5$



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9. Graph the following function, drawing its asymptotes as dashed lines (also, give the equation of each asymptote).

$$g(x) = -1 + \frac{1}{(x+2)^3}$$

SOLUTION:
Asymptotes: $x = -2$ (vertical) and $y = -1$ (horizontal).
PUT GRAPH ....

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10. Solve the equation $x^3 - 7x^2 + 7x + 15 = 0$ given that 3 is a zero.

$$\begin{array}{r l}
x-3 \,\big)\, \overline{x^3 - 7x^2 + 7x + 15} & \quad x^2 - 4x - 5 \\
\underline{x^3 - 3x^2} & \\
-4x^2 + 7x & \\
\underline{-4x^2 + 12x} & \\
-5x + 15 & \\
-5x + 15 &
\end{array}
\Rightarrow
\begin{array}{l}
x^3 - 7x^2 + 7x + 15 = 0 \\
(x-3)(x^2 - 4x - 5) = 0 \\
(x-3)(x-5)(x+1) = 0 \\
x = \boxed{3, 5, -1}
\end{array}$$

11. For each function, find its domain and range.

(a) $h(x) = 3 + \sqrt{7x - 1}$

(b) $f(x) = -6 + \ln(5x + 2)$

SOLUTION:

(a) For the domain, solve $7x - 1 \geq 0$: $x \geq \frac{1}{7}$, so the domain is $[\frac{1}{7}, +\infty)$.

For the range, we note that $h(x)$ is the square root function $\sqrt{x}$ (with range $[0, +\infty)$) with some horizontal modification (not relevant to the range), but shifted up 3, so the the range is $[3, +\infty)$

(b) For the domain, solve $5x + 2 > 0$: $x > \frac{-2}{5}$, so the domain is $(\frac{-2}{5}, +\infty)$.

The range is $(-\infty, +\infty)$, since the starting function $\ln(x)$ has range $(-\infty, +\infty)$, and moving it horizontally and vertically does not change this.