BRONX COMMUNITY COLLEGE
of the City University of New York

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

MTH 30 — Exam 1
Kerry Ojakian — FALL 2025
YOUR NAME (first, then last):

> **Directions:** Write your responses in the provided space. To get full credit you **must** show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. **No** electronic devices are allowed (i.e. no calculators, no phones, no smart watches, etc) - using one during the exam will result in at least a failure on this test. Each question is worth 8 points.

1. Consider the following 3 relations. For each one, is it a function or not? If it is a function, is it injective?

   (a) $\{(0,5),(5,0),(3,1),(-5,1)\}$

   (b) $\{(7,2),(2,3),(4,7),(5,0)\}$

   (c) $\{(1,3),(2,4),(1,7)\}$

   SOLUTION:
   a) Yes it's a function since no repeated input. No it is not injective, since there is repeated output (the 1).
   b) Yes it's a function since no repeated input. Yes it is injective, since there is no repeated output.
   c) No it's a function since there is repeated input (the 1).

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2. What are the absolute extrema of the graph of $y = (x+3)^2 - 1$? (give the x and y coordinates)

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3. Let $g(x) = 3x^3 - x^4$. Evaluate $g(2)$ and $g(-2)$.
   SOLUTION:
   $g(2) = 3(2)^3 - (2)^4 = 3 \cdot 8 - 16 = 8$
   $g(-2) = 3(-2)^3 - (-2)^4 = 3 \cdot (-8) - 16 = -40$

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4. Evaluate $f(0)$, $f(3)$, and $f(4)$ where $f(x) = \begin{cases} x^2 & if\ x \leq 3 \\ 10+x & if\ x > 3 \end{cases}$

   Also, is f a one-to-one function?
   SOLUTION:
   $f(0) = (0)^2 = 0$ (since $0 \leq 3$)
   $f(3) = (3)^2 = 9$ (since $3 \leq 3$)
   $f(4) = 10 + (4) = 14$ (since $4 > 3$)
   No, the function is NOT one-to-one since $f(2) = f(-2) = 4$, that is we repeat the output 4 (or think: two different inputs have the same output).

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5. Consider the function f given by the following table.

x	-3	-1	0	1	7	9
$f(x)$	9	7	6	0	-3	-5

   (a) Evaluate $f(0)$
   (b) Evaluate $f(-3)$
   (c) For what x does $f(x) = -1$
   (d) Is the function increasing or decreasing?

   SOLUTION:
   a) $f(0) = 6$ and b) $f(-3) = 9$ and c) No x values.
   d) Decreasing, since as the input values go up from -3 to 9, the corresponding output values go down from 9 to -5.

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6. Let $h(t) = -4t$. Evaluate $h(-x)$ and $h(x-2)$.
   SOLUTION:
   $h(-x) = -4(-x) = 4x$
   $h(x-2) = -4(x-2) = -4x+8$

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7. Let $f(x) = \dfrac{x^2 - 25}{-2(x+3)(4x-12)}$.

   (a) When is the function undefined?

   (b) What is the domain of the function?

   (c) Evaluate $f(0)$.

   (d) Which x values make $f(x) = 0$?

SOLUTION:
a) Undefined when $-2(x+3)(4x-12) = 0$ so for either $(x+3) = 0$ or $(4x-12) = 0$, so for $x = -3, 3$
b) All reals except -3 and 3.
c) $f(0) = \dfrac{(0)^2 - 25}{-2((0)+3)(4(0)-12)} = \dfrac{-25}{-2(3)(-12)} = -\dfrac{25}{72}$
d) The output is zero when $x^2 - 25 = 0$. To solve, add 25 to both sides to get: $x^2 = 25$, whose solutions are $x = -5, 5$.

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8. Let $f(x) = \dfrac{2}{3x-5}$ and $g(x) = \sqrt{5x-20}$. Find the domain of each of the following.

   (a) $(f+g)$ (b) $(f-g)$ (c) $(f*g)$

SOLUTION:
All 3 functions have the same domain.
First, the domain of f: Undefined when $3x - 5 = 0$, so when $x = \frac{5}{3}$.
Second, the domain of g: Need $5x - 20 \geq 0$, so when $x \geq 4$.
Answer: Take only the numbers in both domains. Since $\frac{5}{3} < 4$, we just take what is allowed by g (i.e. $x \geq 4$) and that exludes the one undefined value for f (i.e. $\frac{5}{3}$) so the domain of each of the 3 functions is: All real x such that $x \geq 4$.

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9. Let $f(x) = x^2 - 4$ and $g(x) = \dfrac{3}{x+1}$. Find the domain of each of the following.

(a) $\dfrac{f}{g}$

(b) $\dfrac{g}{f}$

SOLUTION:
First note that the domain of f: All reals; and domain of g: All reals except -1. Also, in both (a) and (b), note that we at least need both f and g defined, so consider: All reals except -1. Now have the extra worry about division by zero in each case.

a) The bottom function g can be undefined but never zero, so the domain is: All reals except -1.

b) We check when the bottom function can equal zero, that is $x^2 - 4 = 0$. Solve that by adding 4 to both sides to get $x^2 = 4$, so $x = -2, 2$. Thus in this case the function is undefined for $-2, -1, 2$, so the domain is: All reals except $-2, -1, 2$.

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10. Let $f(x) = 2x - 1$ and $g(x) = 5x + 3$.

(a) Find $(f \circ g)(-1)$

(b) Simplify $f(g(x))$

(c) Simplify $(g \circ f)(x)$

SOLUTION:
a) $(f \circ g)(-1) = f(g(-1)) = f(5(-1) + 3) = f(-2) = 2(-2) - 1 = -5$

b) $f(g(x)) = f(5x + 3) = 2(5x + 3) - 1 = 10x + 6 - 1 = 10x + 5$

c) $(g \circ f)(x) = g(f(x)) = g(2x - 1) = 5(2x - 1) + 3 = 10x - 5 + 3 = 10x - 2$

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11. Let $h(x) = (x - 3)^2 + 1$.

(a) On one axis, graph the basic function that $h(x)$ is transformed from (right below here).

(b) Then on another axis, graph $h(x)$ using graph transformations (right below here).

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12. Let $f(x) = \sqrt{x}$ and $g(x) = \dfrac{4}{x^2}$. Answer the following using interval notation.

    (a) Find the domain of $f \circ g$.

    (b) Find the domain of $g \circ f$.

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13. Write a formula for the function obtained when the graph of $f(x) = \sqrt{x}$ is shifted up 2 units and to the right 3 units.
    SOLUTION:
    $g(x) = 2 + \sqrt{x-3}$ since adding outside is a vertical shift (up for positive), while adding inside is a horizontal shift (right for negative).

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14. Consider the function graphed below. For all answers, choose the nearest integer value (example: if an x coordinate looks like about 9.1, just take it to be 9).

    (a) Find the intervals where it increases.

    (b) Find the intervals where it decreases.

    (c) Find all local maxima (both x and y coordinates).

    (d) Find all local minima (both x and y coordinates).

    (e) Find all the absolute extrema (both x and y coordinates).