MTH 30 LECTURE NOTES (Ojakian)

Topic 19: Angles and Radians

OUTLINE
(References: 5.1)

1. Angles
2. Radian Measure
3. Arclength Formula
4. Sector Area Formula

---

1. Angles
   (a) What is an angle?
   (b) Recall **degrees**.
2. Radian measure
   (a) Angles can be measured in degrees or radians. Just different units (like Kilograms versus Grams)
      i. Remember: 180 degress $= \pi$ radians
      ii. Degree to Radian: Mult by $\pi/180$
      iii. Radian to Degree: Mult by $180/\pi$
   (b)

   PROBLEM 1.

      i. Convert 90° *to radians.*
      ii. Convert $\pi/3$ *radians to degrees.*

   (c) Why radian measure preferred? ...

3. Arc length Formula and Sector Area Formula

Question: What is the circumference of the earth? How can we measure this with a stick and some common sense??

(a) Terminology:

i. A central angle in a circle

ii. A central angle and its corresponding **arc**

iii. A central angle and its corresponding **sector**

(b) The Formulas

i. **Must have θ measured in radians!**

ii. Arclength formula: $s = r\theta$

Think: $s = (2\pi r)\left(\dfrac{\theta}{2\pi}\right) = r\theta$

iii. Sector formula: $A = (\theta r^2)/2$

Think: $A = (\pi r^2)\left(\dfrac{\theta}{2\pi}\right) = (\theta r^2)/2$

(c) Typical Use: Given 2 of the quantities, find the third quantity.

PROBLEM 2. *Suppose a circle with radius 5 has a central angle of* $\pi/4$. *How long is the arc of the circle that corresponds to this central angle?*

PROBLEM 3. *Suppose a circle has a central angle of* 90 *degrees which subtends an arc of length* 10. *Find the radius of the circle.*

PROBLEM 4. *Suppose a sector of a circle has a central angle of* $2\pi/5$ *and the radius of the circle is 2. Find the area of the section.*

PROBLEM 5. *Suppose a sector of a circle has area* 12π *and the central angle is* 90^o. *Find the* area *of the circle.*

(d) The circumference of the earth (reference: Trigonomentric Delights)

i. Measured by ancient Greek scientist Eratosthenes in 240 BC.

ii. Two locations: 1) Summer solstice at noon when sun directly overhead (like Aswan, where the light reaches bottom of a well), and 2) Some other place, like Alexandria.

iii. Measure the arclength between the two towns: 5000 Stadia (one stadium is between 607 and 738 feet).

iv. Measure the central angle. How?? ...

v. Put a stick in the ground in Alexandria and measure the rotation required to lose the shadow (7.2 degrees)

vi. Current science gives the correct value at about 24,900 miles.