MTH 28.5 LECTURE NOTES (Ojakian)

Topic 31: Complex Numbers

OUTLINE
(References: 8.8)

1. Complex Numbers: Addition, Subtraction, Multiplication, Division
2. Fundamental Theorem of Algebra

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1. Solving Unusual Equations

PROBLEM 1. *Solve each equation.*

(a) $x^2 - 1 = 0$
(b) $x^2 + 1 = 0$

Definition 1. *The number* $\mathbf{i}$ *is a new number such that* $\mathbf{i}^2 = -1$.

PROBLEM 2. *Solve each equation (complex number solutions allowed).*
Can do by "Square Root Property"

(a) $x^2 - 100 = 0$
(b) $x^2 + 100 = 0$

Observe: Some polynomial equations have solutions, some have none (if only real numbers allowed!).

Theorem 1. *(Fundamental Theorem of Algebra) Every polynomial equation has a solution if we allow complex solutions.*

2. Complex Numbers: Powers of i and square roots of negatives

PROBLEM 3. *Simplify each expression.*

(a) $\mathbf{i}^3$
(b) $\mathbf{i}^4$
(c) $\mathbf{i}^5$
(d) $\mathbf{i}^6$
(e) $\mathbf{i}^{1001}$ *(for fun, if you like ...)*
(f) $\sqrt{-1}$
(g) $\sqrt{-4}$
(h) $\sqrt{-8}$

3. Complex Numbers: Addition and Subtraction

PROBLEM 4. *Simplify each expression.*

(a) $4 + 7\mathbf{i} + 10 - 3\mathbf{i}$

(b) $(2 + 3i) - (-1 - 7i)$

(c) $3\mathbf{i} + 7 - 5\mathbf{i} + 3 + 2\mathbf{i}^2$

(d) $-2\mathbf{i}^4 + 2 - 5\mathbf{i} + 3 + 2\mathbf{i}$

Definition 2. *A* ***complex number*** *(in "standard form") is a number of the form* $a + b\mathbf{i}$ *where* a *and* b *are real numbers.*

4. Complex Numbers: Multiplication

PROBLEM 5. *Perform the operation and simplify.*

(a) $7\mathbf{i} \cdot 10\mathbf{i}$

(b) $3\mathbf{i}(5 - 2\mathbf{i})$

(c) $(2 + 3\mathbf{i})(-1 - 7\mathbf{i})$ *(notice how this question is different from Problem 4, part (b)).*

5. Complex Numbers: Division

For a simplified complex number, we do not want $\mathbf{i}$ on the bottom of a fraction.

PROBLEM 6. *Perform the operation, simplify, and write in standard form.*

(a) $\dfrac{5}{\mathbf{i}}$

(b) $\dfrac{6}{9\mathbf{i}}$

(c) $(3 + \mathbf{i})(3 - \mathbf{i})$ *(called "conjugates")*

(d) $\dfrac{1}{7 - 2\mathbf{i}}$

(e) $\dfrac{2}{5 + \mathbf{i}}$