MTH 28.5 LECTURE NOTES (Ojakian)

Topic 12: Lines

OUTLINE

References: 3.1, 3.2

1. Linear versus non-linear equations
2. Lines
3. Intercepts and Slope

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1. Graphing Lines

PROBLEM 1. *For each equation, guess which ones have a graph that is a line, and which ones have a graph that is* ***not*** *a line.*

(a) $y = -x$

(b) $y - 3 = x^2$

(c) $2y = 2 + x$

(d) $y - 3x^2 = -10$

PROBLEM 2. *Can you think of a method for determining which equations have a line for a graph?*

2. Linear Equations

Definition 1. *An equation in two variables (x and y) is called* ***linear*** *if can be simplified to the form: "$y = Mx + B$" or "$x = C$" or "$y = C$"*

Definition 2. *(*Intuitive*) An equation in two variables (x and y) is called* ***linear*** *if has only "x" terms, "y" terms, and numbers, with everything else cancelling out.*

PROBLEM 3. *Consider the equations we have looked at so far. Which ones are linear? Which ones are non-linear?*

Theorem 1.

- *The graph of a linear equation is a line.*
- *Any line can be described with some linear equation.*

3. Intercepts

(a) Recall the definition of $x-$intercepts and $y-$intercepts.

PROBLEM 4. *For each equation, find the intercepts of its graph.*

i. $y = 3x + 3$

ii. $x + 2y = 1$

(b) To find intercepts:

i. To find $x-$intercept, set $y = 0$, then solve for x.
ii. To find $y-$intercept, set $x = 0$, then solve for y.

4. Graphing a line

PROBLEM 5. *Consider problem 4. Graph each of the equations.*

(a) To graph a line:
   i. Find any two points (such as x and y intercepts).
   ii. Connect the points by a straight line

5. Special Lines

PROBLEM 6. *Graph each equation* **in the plane**.

(a) $y = 3$
(b) $x = 2$

***PROBLEM* 7.** *Describe a method for graphing lines like the ones appearing in the last two problems.*

6. Slope

Slope: A number that measures the "steepness" of a line.

(a) Finding slope by "lining up the points"
   i. Line up points and subtract
   ii. Get the $y-$change
   iii. Get the $x-$change.
   iv. Slope $= \dfrac{y - \text{change}}{x - \text{change}}$
   v. Note: Be careful of sign!

(b)

PROBLEM 8. *Suppose a line contains the following points:* $(0, 1), (2, 5), (-1, -1)$. *Find its slope.*

Theorem 2. *The slope of a line is the same, no matter what two distinct points are used to compute it.*

PROBLEM 9. *Find the slope of the line* $x + y = 3$

***PROBLEM* 10.** *Using problems as examples, answer the following questions.*

i. What is the slope of a horizontal line?
ii. What is the slope of a vertical line?
iii. Describe the difference between a line with positive slope versus a line with negative slope.

7. Slope-Intercept Form of a line

***PROBLEM* 11.** *Based on the above problems, guess a fast way to determine the slope of a line.*

(a) Line with slope m and y-intercept b has equation $y = mx + b$.

(b) Equation $\rightarrow$ slope and y-intercept

i. Put in slope-intercept form (i.e. solve for y)

ii. Then find m and b.

(c) Using Slope-Intercept Form to Graph a line

i. Use the y-intercept as one point.

ii. Use the slope to find a second point:

A. Write slope as $\frac{(+ \text{ or } -)\ V}{H}$

B. Start at the y-intercept

C. Move to the right H

D. Move up or down V (up for positive slope, down for negative)