MTH 28.5 LECTURE NOTES (Ojakian)

Topic 11: Equations in 2 Variables and The Plane

OUTLINE

References: 3.1

1. Equations in 2 variables
   (a) Checking a point
   (b) From one variable → Other variable
   (c) Finding a solution
2. Cartesian Plane
   (a) Coordinates → Point
   (b) Point → Coordinates
3. Different ways to represent a relationship between 2 quantities
   (a) By an equation
   (b) By a picture - GRAPHING!
4. Real-World applications of these ideas

---

1. Equations in two variables

(a) Checking a Pair

PROBLEM 1. *Consider the equation $x^2 + y^2 = 4$. Which of the following pairs of values make the equation true?*

i. $x = 2$ *and* $y = 0$

ii. $x = 1$ *and* $y = 3$

iii. $x = 0$ *and* $y = -2$

Definition 1. *Given an equation in two variables, a* ***solution*** *is a pair of numbers that make the equation true.*

Definition 2. *Given an equation in two variables, its* ***solution set*** *is the set of all its solutions.*

PROBLEM 2. *Find three solutions to the equation $x^2+y^2 = 4$ (recall problem 1).*

(b) Convention: For an equation in variables "x" and "y", the solutions are listed as pairs of numbers: The first number represents "x" and the second number represents "y".

PROBLEM 3. *Represent three solutions to the equation $x^2 + y^2 = 4$ using the "pair convention".*

PROBLEM 4. *Consider the equation* $2x^3 - y^2 = 7$. *Check if each pair is a solution.*

i. $(2, -3)$

ii. $(-1, 0)$

(c) Finishing a Pair

PROBLEM 5. *Consider the equation* $2x + 3y = 5$. *Find the solution which has* $x = -2$, *also the one that has* $y = -2$

(d) Strategy to "finish a pair"

i. Solve for the other variable

ii. Plug in given value

iii. (Can reverse the order)

(e) Finding some solutions

PROBLEM 6. *Find* **any** *three solutions to the following equation:* $2y - x = 1$.

(f) Strategy for finding solutions

i. Choose one variable as you please!

ii. Plug it in and solve for the other variable.

2. Cartesian Plane

Two Goals:

(a) Represent data as a picture.

(b) Represent an equation in two variables using a picture.

Definition 3. *The* ***Cartesian Plane*** *consists of two number lines: the x-axis and the y-axis.*

(a) Two skills:

i. Given the coordinates of a point, plot it.

ii. Given the plotted point, find its coordinates

PROBLEM 7. *Draw the x and y axis. Then plot the following points (label each point by the letter).*

A. $(2, 5)$

B. $(5, 2)$

C. $(-1, 2)$

D. $(3, -2)$

E. $(-4, -3)$

F. $(0, 4)$

G. $(4, 0)$

(b) Terminology

i. Origin

ii. 4 quadrants

***PROBLEM* 8.** *Consider each point below in the above problem. If it is on an axis, state which axis it is on (x-axis or y-axis?). Otherwise, state which quadrant it is in. Can do the same for the above points.*

(c) Application to real world data

PROBLEM 9. *Read pages 49, 50 in the* Real World Book*, and plot (maybe just last 5 years).*

i. Choose step size (for each axis) that meets the needs

ii. Mark every increment by that step size

3. Graphing

Definition 4. *A* ***graph*** *is a set of points on the plane.*

PROBLEM 10. *Draw the $x-$axis and the $y-$axis. Then do the following.*

(a) Draw any graph that contains a **finite** *number of points.*

(b) Draw any graph that contains an **infinite** *number of points.*

Definition 5. *Given an equation in two variables,* ***the graph of the equation*** *is the graph consisting of all the points in the solution set of the equation.*

PROBLEM 11. *Estimate the graph of each equation by plotting some solutions and guessing what the rest of the solutions are. Are either of the graphs lines?*

(a) $3 - y = x$

(b) $x^2 + y^2 = 16$

Definition 6. *Consider any graph.*

- *Its $x-$****intercepts*** *are the points where it touches the $x-$axis.*
- *Its $y-$****intercepts*** *are the points where it touches the $y-$axis.*

PROBLEM 12. *Consider problem 11. For each graph, what are the $x-$intercepts and what are the $y-$intercepts?*