

MTH 28.5 LECTURE NOTES (Ojakian)

Topic 10: Solving Inequalities

OUTLINE

References: 2.5

1. Inequality terminology
 - (a) Basic terminology
 - (b) Solution set
 - (c) Graphing inequalities
 2. Solving Linear Inequalities
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1. Meaning of inequality signs

PROBLEM 1. For each statement, is it true or false?

(a) $5 < 6.3$

(b) $0 > \frac{91}{13}$

(c) $-43 > -100$

(d) $5 = 5.03$

(e) $3 \leq 7$

(f) $3 < 3$

(g) $3 \leq 3$

(h) $-\frac{3}{4} \geq -10$

(i) $-\frac{3}{4} \geq -\frac{3}{4}$

(j) $-\frac{3}{4} < -\frac{3}{4}$

(k) $-\frac{3}{4} > -\frac{3}{4}$

2. Solution set

Definition 1. The *solution set* of an inequality is the set of numbers that make it true.

PROBLEM 2. For each inequality, find three different numbers in its solution set.

(a) $x > -5$

(b) $x \geq -5$

(c) $2u + 1 \leq u$

Definition 2. To *solve* an inequality means to find **all** the numbers in its solution set.

3. Graphing solutions on the number line

How do you represent **all** the solutions? Often is infinite!

PROBLEM 3. For each inequality, graph its solution set on the number line.

- (a) $x > 3$
- (b) $-1 > x$
- (c) $-5 \leq x$

Definition 3.

- The symbols $<$ and $>$ are called **strict inequalities**. To graph them, use an open circle or a round brace.
- The symbols \leq and \geq are called **non-strict inequalities**. To graph them, use a filled-in circle or a square brace.

4. Solving Inequalities

- (a) Principle: Use all the usual rules (for equations), **except for the exceptions ...**
- (b)

PROBLEM 4. For each inequality graph its solution set.

- i. $x \geq 5$
- ii. $2x \geq 10$
- iii. $-2x \geq -10$
- iv. $-2x \leq -10$

PROBLEM 5. Consider problem 4 and the first inequality $x \geq 5$.

- i. Describe how we change $x \geq 5$ to get the second inequality $2x \geq 10$? (i.e. what did we do to both sides of the inequality $x \geq 5$?)
- ii. Describe how we change $x \geq 5$ to get the third inequality $-2x \geq -10$?
- iii. Describe how we change $x \geq 5$ to get the fourth inequality $-2x \leq -10$?
- (c) Principle for solving inequalities: Use all the usual rules (for equations), **except** that when you multiply or divide by a negative number **reverse the inequality**.
- (d)

PROBLEM 6. Solve each inequality

- i. $10x > 50$
- ii. $-4x \leq 20$
- iii. $\frac{x}{2} \geq 3$
- iv. $\frac{x}{-3} > 6$

PROBLEM 7. From section 2.5 do exercises: 314, 316

5. Possible kinds of solutions

PROBLEM 8. *Using common sense (not algebra), find the solution set of each inequality.*

(a) $x < x - 5$

(b) $x + 2 \geq x$

PROBLEM 9. *Consider the examples in problem 8.*

(a) *If you try to solve them using algebra, what happens?*

(b) *Describe a rule for determining when there is **no solution** and when **all real numbers are solutions**.*

PROBLEM 10. *For each of the following inequalities, it either has **no solutions** or **all real numbers as solutions**. Determine which is the case for each inequality.*

(a) $2x + 3x \leq 5(x + 10)$

(b) $-2(-3x + 1) > x + 5(2 + x)$