MTH 28.5 LECTURE NOTES (Ojakian)

Topic 7: Solving Equations

OUTLINE
References: 2.1

1. Theoretical Background
   (a) Solution Sets
   (b) What it means to solve an equation
2. Solving Equations
   (a) By Guess-and-Check
   (b) Using Algebraic Tools

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1. Solution Sets and Solving

Definition 1. *Given an equation, its* ***solution set*** *is the set of numbers that make it true.*

PROBLEM 1. *For each equation, find its solution set. Then determine which are "identities" and which are not*

(a) $x + 5 = 15$
(b) $x + 2x = 3x$
(c) $y + y = y$
(d) $x = x + 1$
(e) $x^2 = 25$

Theorem 1. *Given an equation, there are three possibilities:*

(a) It is always true (i.e. it is an identity), or
(b) It is sometimes true, or
(c) It is never true (has "No Solution")

Definition 2. *To* ***solve*** *an equation means to find the solution set of the equation.*

2. Two Uses of Equality Sign

(a) Previously, for simplifying and evaluating:
Between expressions that are ALWAYS equivalent.
(b) Now, for equations:
Between ANY two expressions, not necessarily equivalent.
(c) PLEASE: Only use the equality sign for one of the above two purposes!

3. Solving Equations by Guess-and-Check

(a) Checking a possibility.

PROBLEM 2. *Consider the equation* $x^2 = x - x^2$.

i. Is $x = 0$ *a solution?*

ii. Is $x = -1/3$ *a solution?*

iii. Is $x = 1/2$ *a solution?*

(b) Guessing and Checking

PROBLEM 3. *Consider the equation* $x + 3 = -5$. *Solve it by guessing and checking.*

4. Tools for solving equations

PROBLEM 4. *Find the solution set of the following equation:*

$$4x^2 = 36$$

Then find the solution set of each equation below.

(a) $4x^2 + 2 = 38$

(b) $4x^2 - 2 = 34$

(c) $40x^2 = 360$

(d) $2x^2 = 18$

PROBLEM 5. *Using the examples from the last problem describe various ways that you can change an equation without changing its solution set.*

PROBLEM 6. *Use the tools from above to solve the following equations.*

(a) $x - 10 = 473$

(b) $10x = 451$

(c) $u + 13 = 174$

(d) $u/5 = 21$

***PROBLEM* 7.** *Complete the following phrases:*

(a) To remove an addition do ...

(b) To remove an subtraction do ...

(c) To remove a multiplication do ...

(d) To remove a division do ...

5. More complicated equations

Same idea, just a little more algebra ...

PROBLEM 8. *Solve each equation, writing a justification for each new line in your solution.*

(a) $13x + 7 - 10x = -2 - 3$

(b) $4x - 6 = 7x - 3$

(c) $2(3 - 2x) = 2 - (3x - 4)$

PROBLEM 9. *From the textbook, section 2.1, do some from among 5 to 30, 43 to 64.*

(a) Strategy for solving

i. Expand expressions using distributivity
ii. Simplify each side by combining like terms
iii. Use addition and subtraction to collect the variable on one side and numbers on the other side
iv. Multiply or divide to isolate the variable

(b) Ideal way to write your answer

i. Top line is the original equation
ii. Bottom line is of the form $x = \ldots$
iii. An intermediate line is derived from the one directly above it, with an explanation of how it was obtained.

6. Possible kinds of solutions

Identity, Contradiction, or Conditional

PROBLEM 10. *Solve each equation (if something unusual happens when trying to solve the equations, try to explain what is happening).*

(a) $5x + 3 = 5x + 3$

(b) $5x + 3 = 5x + 1$

PROBLEM 11. *Based on the examples in the last problem complete the following phrases. When applying algebriac operations to an equation and ...*

- *You arrive at an equation which is ALWAYS TRUE, then ... [*Fill It In*].*
- *You arrive at an equation which is ALWAYS FALSE, then ... [*Fill It In*].*