MTH 28.5 LECTURE NOTES (Ojakian)

Topic 3: Arithmetic with Fractions

OUTLINE

References (1.2, .1.3, 1.4)

1. Multiplying and Dividing Fractions/Decimals
2. Adding and Subtracting Fractions/Decimals
3. Prime and composite numbers
4. Factoring integers
5. Least Common Multiple (LCM)

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1. Factoring Integers

(a) Prime and composite numbers

Definition. *An integer N is **prime** if $N > 1$ and the only positive integers dividing into N evenly are 1 and N.*

PROBLEM 1. *For each number, determine whether it is 1) prime, 2) not prime*

$$5,\ 4,\ 2,\ 9,\ 1,\ 10000,\ 51$$

(b) Factoring an integer

Definition. *An integer is **factored** if it is written as a product of primes (*the factorization of a prime number is just the number itself*).*

Theorem. *A positive integer larger than 1 has exactly one factorization (except for re-ordering).*

PROBLEM 2. *Factor each of the following integers:* $10,\ 63,\ 17$.

Note: Use exponents to abbreviated repeated multiplication.

2. The Fundamental Principle of Fractions

(a) $\dfrac{a}{b} = \dfrac{ac}{bc}$ (multiply top and bottom by same number)

(b) $\dfrac{a}{b} = \dfrac{a \div c}{b \div c}$ (divide top and bottom by same number)

PROBLEM 3. *Write each fraction as 3 different fractions:* $2/4,\ 7/3,\ -1/4$.

3. Simplified Fraction (or Reduced Fraction)

Definition. *A fraction a/b is **simplified** if there is no integer larger than 1 divides evenly into both a and b.*

PROBLEM 4. *Simplify the fractions:* $28/36,\ 3/13$

PROBLEM 5. *Simplify the fraction using factoring as a tool:* $98/56$

4. Multiplication of Fractions
   (a) Multiply tops
   (b) Multiply bottoms
   (c) Simplify

   PROBLEM 6.

   i. Compute the following: $\frac{1}{2} \cdot \frac{1}{5}$

   ii. Convert the fractions to decimal and then try the calculation.

   iii. Does the answer make sense? (it should!)

   PROBLEM 7. *Compute the following:*

   i. $\frac{2}{3} \cdot \left(-\frac{1}{5}\right)$

   ii. $\frac{-2}{-3} \cdot \left(\frac{-1}{5}\right)$

   iii. $\frac{2}{5} \cdot \frac{15}{8}$

   iv. $\frac{49}{88} \cdot \frac{-11}{7}$

5. Dividing Fractions
   (a) Find reciprocal of second fraction
   (b) Then multiply

   PROBLEM 8.

   i. Compute the following: $\frac{3}{2} \cdot \frac{1}{4}$

   ii. Convert the fractions to decimal and then try the calculation.

   iii. Does the answer make sense? (it should!)

   PROBLEM 9. *Compute the following:*

   i. $-\frac{50}{12} \div \frac{-5}{3}$ *(Simplify before calculating!)*

   ii. $\frac{1}{5} \div \frac{10}{7}$ *(Be careful on simplifing!)*

6. Adding and Subtracting Fractions
   (a) When denominators are the same.
   Recall interpretation: A/B means to take A steps of size $1/B$ each.

   PROBLEM 10. *By* **counting on the number line** *compute the following:*

   i. $\frac{3}{4} + \frac{2}{4}$

ii. $-\frac{3}{4}+\frac{-1}{4}+\frac{5}{4}$

iii. Convert the fractions to decimals and try it. Make sense? (it should!)

7. Conversions

   (a) Fraction → Mixed

   Divide - Use quotient as integer part and use remainder as the top

   PROBLEM 11. *Convert* 13/3 *to a mixed number (use both methods).*

   (b) Mixed → Fraction

   i. Find the product of the integer part and the bottom

   ii. Add to the top

   PROBLEM 12. *Write* $3\frac{2}{7}$ *as an improper fraction. What happens if it were negative?*

   (c) Fraction → Decimal: Just Perform division

   PROBLEM 13. *Write the fractions as decimals (rounded to nearest tenth):* $3/2, -1/3$.

   (d) Decimal → Fraction

   i. Write as "(decimal)/1"

   ii. Multiply top/bottom by appropriate power of 10 (so decimal point is gone)

   iii. Then simplify

   PROBLEM 14. *Convert the decimal* 2.8 *to a fraction.*

8. Denominators NOT the same when Adding and Subtracting Fractions

   (a) Method.

   i. Make the denominators the same! (i.e. find a **common denominator**)

   ii. Then add or subtract tops

   iii. Leave the bottom the same

   PROBLEM 15. *Compute the following:*

   A. $\frac{7}{20}+\frac{2}{5}$

   B. $\frac{1}{-2}-\frac{-2}{3}+-\frac{2}{5}$

9. Quick way to get a common denominator

   (a) Just multiple all the bottoms together to get the new bottom! (may get large numbers!)

   (b) Thus: Multiply each fraction's top and bottom by the *other* bottoms

10. To find LEAST common denominator

    (a) Least Common Multiple

    Definition. *The* ***Least Common Multiple*** *(* ***L.C.M.****) of a group of integers is the smallest integer that all the integers in the group divide into evenly.*

    i. Easier Way to find LCM:
    Count by multiples till the first common multiple is reached.

PROBLEM 16. *Find the LCM for each group of integers.*

A. 3, 10

B. 4, 6

C. 120, 10, 5

D. 24, 18 *(Too hard? See next method.)*

ii. Harder Method for finding LCM (*useful if the numbers are larger*).
Nice Picture: Page 48 in book.

A. Factor each integer in the group

B. Make a list of all the primes that appear.

C. For each prime, take the largest occurence

D. The LCM = product of these prime powers.

iii.

PROBLEM 17. *Find the LCM for each group of integers.*

A. 24, 18

B. 180, 225, 15

(b) Addition/Subtraction with large denominators

Definition. *Given a group of fractions, the* ***Least Common Denominator*** *(* ***L.C.D.*** *) is the LCM of the denominators.*

The LCD is the smallest common denominator that can be used for the addition and subtraction of fractions.

PROBLEM 18. *Compute the following:*

i. $\frac{2}{125} + \frac{1}{75}$

ii. $\frac{1}{30} - \frac{-6}{-84} + -\frac{7}{30}$

11. Applications

PROBLEM 19. *Find the area and perimeter of a rectangle of width 7 amd height 3/14.*

PROBLEM 20. *If I pay every student in class 5.25 to study, how much do I lose? And if I only have 30 dollars, how many students can I pay?*