MTH 28.5 LECTURE NOTES (Ojakian)

Topics 1: Numbers-Properties

OUTLINE
References: 1.2, 1.3, 1.4

1. History

2. Number Line

3. Numbers: Reals, Integers, Rationals, Irrationals.

4. Rationals

5. Negative Numbers

   (a) Opposites

   (b) Absolute Value

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1. <u>History Questions</u> (see file "History of Numbers")

   (a) When and where do we find humans first using numbers?

   (b) When and where were some important kinds of numbers discovered? (zero, negative numbers, fractions)

   (c) Are there any other numbers?

2. <u>Kinds of Numbers</u>

   Definition. *A number is an **integer** if it is a whole number, **including** zero and the negative whole numbers.*

   Definition. *A number is **rational** (i.e. a "fraction") if it can be written as a quotient of two integers.*

   Definition. *A number is **irrational** if it <u>cannot</u> be written as a fraction of integers.*

3. <u>Rational Numbers</u>

   (a) Three ways to represent rational numbers

      i. Mixed Number (integer part **plus** fractional part)

      ii. Decimal

      iii. Quotient: Form A/B

         A. Perspective: A/B means to take A steps of size $1/B$ each.

         B. Called **Improper Fraction** if $A > B$

      iv.

      PROBLEM 1.

      A. *How many $1/3$'s does $5/3$ represent? Place it on the number line.*

      B. *How many $1/3$'s does $8/3$ represent? Place it on the number line.*

    C. How many 1/2's does 7/2 represent? Place it on the number line.

    D. Which is largest?

4. <u>Number Line.</u>

Definition. *The **number line** is a horizontal line going infinitely far to the right and left with the following properties:*

  (a) *Zero is in "middle"*

  (b) *Positive numbers to right of zero*

  (c) *Negative numbers to left of zero*

  (d) *Left is smaller. Right is larger.*

PROBLEM 2.

  (a) *Draw the number line and place the following numbers on it:*

$$0, \ 4, \ 7, \ -5, \ -8, \ 1/2, \ -1/2, \ 11.75, \ -7.75, \ 11\frac{1}{3}, \ -3\frac{3}{4}$$

  (b) *Which is the largest number and which is the smallest number?*

5. <u>Inequalities</u>

  (a) Strict

  (b) Non-strict

6. <u>Uses of negative numbers</u>

  (a) Temperature

  (b) Above/below sea level

  (c) Profit/loss

    PROBLEM 3.

     i. *The lowest natural temperature ever directly recorded at ground level on Earth is -89.2 Celsius, which was at the Soviet Vostok Station in Antarctica, in 1983.*

     ii. *The absolute coldest temperature in the universe is -270.45 Celsius.*

     iii. *Place the two temperatures on the number line.*

     iv. *Which temperature is a smaller number? Which represents a colder temperature?*

7. <u>Opposite (or negation) of number</u>

Definition. *The **opposite** of a number is the other number that is the same distance from 0.*

Definition. *(Alternative) To find the **opposite** of a number, if the number is positive, put a negative sign in front of it; if a number is negative, remove the negative sign from in front.*

PROBLEM 4. *Find the opposites of each number:* 14, -2, 99, -112, ,2/5 0

PROBLEM 5. *Simplify each number (i.e. write with as few negative signs as possible):*

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- $--4$
- $----9$
- $---1$
- -0

PROBLEM 6. *Suppose a number has some amount of negative signs in front of it. What is the rule for determining if the number is positive or negative?*

8. <u>Absolute value</u>

   (a) Operation: Takes one number and outputs a positive.

   (b) Distance of a number from 0.

   (c) Never negative!

   PROBLEM 7. *Simplify each:*

   - $|-7|$
   - $|7|$
   - $|--5|$