WORKBOOK. MATH 05. BASIC CONCEPTS OF MATHEMATICS I.

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

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1. A review of fractions

(1) What are the natural numbers?

(2) What are the whole numbers?

(3) What are the integers?

(4) What are the rational numbers? Give five examples.

(5) What are the irrational numbers? Give five examples.

(6) What are the real numbers? Give five examples.

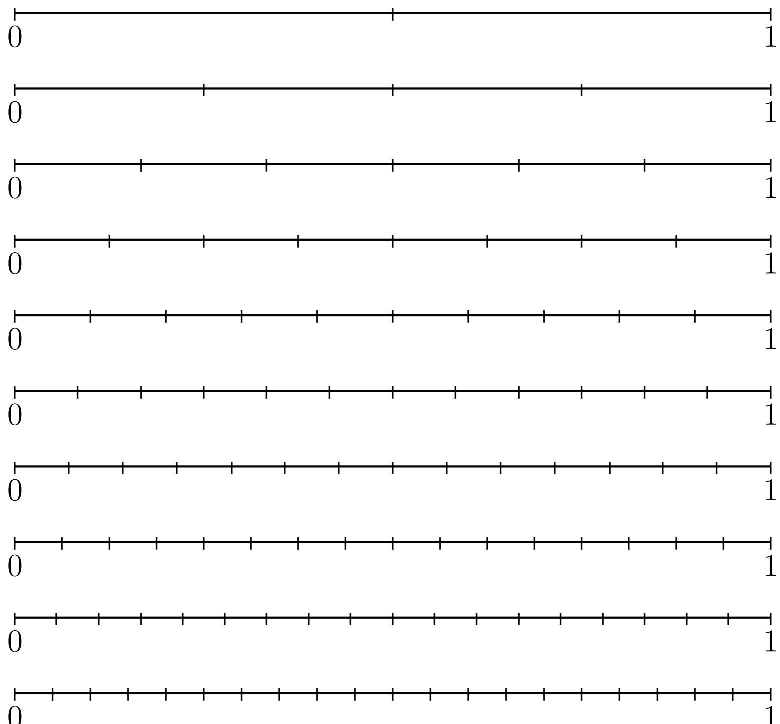
(7) Do you know any numbers which are not real numbers? If so, give five examples.

(8) Draw a diagram illustrating different number systems.

In this section we will work with rational numbers.

(1) Explain the following equalities using the number lines below:

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \text{---} = \text{---} = \text{---} = \text{---}$$



(2) Fill in the blanks and explain using number lines as above:

$$\frac{1}{3} = \frac{\quad}{6} = \frac{3}{\quad} = \frac{\quad}{12} = \frac{5}{\quad} = \frac{\quad}{18} = \frac{7}{\quad} = \frac{10}{\quad} = \frac{\quad}{45}$$

(3) Fill in the blanks:

$$\frac{2}{5} = \frac{4}{\quad} = \frac{\quad}{15} = \frac{10}{\quad} = \frac{14}{\quad} = \frac{\quad}{40} = \frac{\quad}{55} = \frac{24}{\quad} = \frac{\quad}{100}$$

(4) Fill in the blanks:

$$\frac{3}{7} = \frac{6}{14} = \frac{9}{\quad} = \frac{\quad}{28} = \frac{18}{\quad} = \frac{\quad}{49} = \frac{\quad}{70} = \frac{36}{\quad} = \frac{\quad}{105} = \frac{300}{\quad}$$

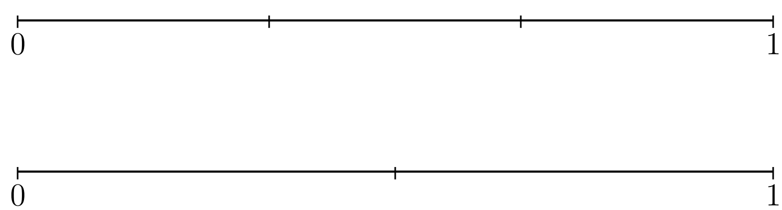
(5) Simplify the following fractions:

$\frac{10}{40}$ $\quad$ $\frac{12}{28}$ $\quad$ $\frac{36}{48}$ $\quad$ $\frac{35}{105}$

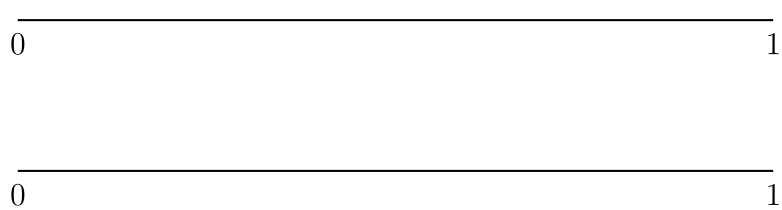
$\frac{11}{66}$ $\quad$ $\frac{120}{160}$ $\quad$ $\frac{45}{55}$ $\quad$ $\frac{85}{15}$

$\frac{100}{45}$ $\quad$ $\frac{40}{160}$ $\quad$ $\frac{65}{75}$ $\quad$ $\frac{85}{105}$

(6) Using the two number lines find the sum $\frac{1}{3} + \frac{1}{2}$.



(7) Using the two number lines find the sum $\frac{3}{4} + \frac{1}{6}$.



(8) Find the least common denominator (do you see why we need this?) for the following denominators:

- 12, 18
- 2, 3, 6
- 10, 20, 30
- 8, 12, 36

(9) Perform the following additions and simplify your answers:

$8+7$ $\quad \frac{8}{15}+\frac{7}{15}$ $\quad \frac{2}{5}+\frac{3}{5}$ $\quad \frac{4}{7}+\frac{10}{7}$

$\frac{10}{11}+\frac{7}{2}$ $\quad \frac{8}{15}+\frac{1}{5}$ $\quad \frac{2}{7}+\frac{3}{5}$ $\quad \frac{2}{9}+\frac{1}{4}$

$\frac{7}{12}+\frac{5}{18}$ $\quad \frac{1}{2}+\frac{7}{3}+\frac{11}{6}$ $\quad \frac{7}{10}+\frac{3}{5}+\frac{11}{3}$ $\quad \frac{3}{4}+\frac{7}{12}+\frac{29}{36}$

(10) Perform the following subtractions and simplify your answers:

$8-7$ $\quad \frac{8}{15}-\frac{7}{15}$ $\quad \frac{2}{5}-\frac{3}{5}$ $\quad \frac{4}{7}-\frac{10}{7}$

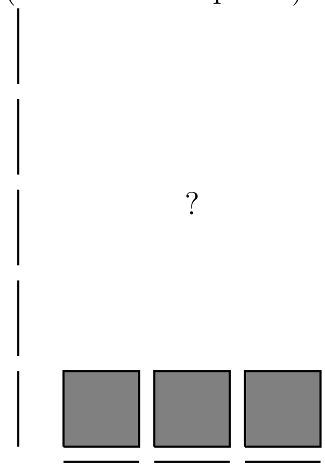
$\frac{10}{11}-\frac{1}{2}$ $\quad \frac{8}{15}-\frac{1}{5}$ $\quad \frac{2}{7}-\frac{3}{5}$ $\quad \frac{2}{9}-\frac{1}{4}$

$\frac{7}{12}-\frac{5}{18}$ $\quad \frac{1}{2}-\frac{7}{3}-\frac{11}{6}$ $\quad \frac{7}{10}-\frac{3}{5}-\frac{11}{3}$ $\quad \frac{3}{4}-\frac{7}{12}-\frac{29}{36}$

(11) Perform the following and simplify your answers:

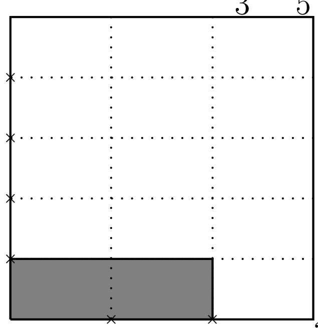
- $1+\frac{1}{2}$
- $1-\frac{1}{2}+\frac{1}{3}$
- $1+\frac{1}{2}-\frac{1}{3}+\frac{1}{4}$
- $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}$

(12) How would you explain 3×5 geometrically/pictorially? (Hint: Think squares).



(13) How would you explain 2×4 geometrically/pictorially? (Hint: Think squares).

(14) If the large square represents one, then what does the shaded region represent? Use this picture to explain $\frac{2}{3} \times \frac{1}{5}$.



(15) Use a square to explain $\frac{3}{4} \times \frac{2}{3}$. Simplify your answer.

(16) How do you multiply two fractions? Do you need common denominators in this case?

(17) Perform the following multiplications and simplify your answer:

8×7 $\qquad \frac{3}{5} \times \frac{4}{7}$ $\qquad \frac{2}{5} \times \frac{3}{5}$ $\qquad \frac{4}{7} \times \frac{10}{7}$

$\frac{10}{13} \times \frac{7}{12}$ $\qquad \frac{8}{9} \times \frac{1}{5}$ $\qquad \frac{2}{7} \times \frac{3}{5}$ $\qquad \frac{2}{9} \times \frac{1}{4}$

$\frac{7}{10} \times \frac{5}{11}$ $\qquad \frac{1}{2} \times \frac{7}{3} \times \frac{11}{6}$ $\qquad \frac{7}{10} \times \frac{3}{20} \times \frac{11}{3}$ $\qquad \frac{3}{8} \times \frac{7}{12} \times \frac{8}{49}$

(18) Answer the following:

- How many threes make fifteen?
- How many 3's make 15?
- Fill in the blank: $3 \times$ ____________ $= 15$.
- What is $15 \div 3$?

(19) Answer the following:

- How many quarters make three-quarters?
- How many $\left(\frac{1}{4}\right)$'s make $\left(\frac{3}{4}\right)$?
- Fill in the blank: $\left(\frac{1}{4}\right) \times$ ____________ $= \left(\frac{3}{4}\right)$.
- What is $\left(\frac{3}{4}\right) \times \left(\frac{4}{1}\right)$?
- What is $\left(\frac{3}{4}\right) \div \left(\frac{1}{4}\right)$?

(20) Answer the following:

- How many two-thirds make eight-thirds?
- How many $\left(\frac{2}{3}\right)$'s make $\left(\frac{8}{3}\right)$?
- Fill in the blank: $\left(\frac{2}{3}\right) \times$ ____________ $= \left(\frac{8}{3}\right)$.
- What is $\left(\frac{8}{3}\right) \times \left(\frac{3}{2}\right)$?
- What is $\left(\frac{8}{3}\right) \div \left(\frac{2}{3}\right)$?

(21) Answer the following:

- Fill in the blank: $\left(\frac{1}{5}\right) \times$ ____________ $= \left(\frac{3}{4}\right)$.
- What is $\left(\frac{3}{4}\right) \times \left(\frac{5}{1}\right)$?
- What is $\left(\frac{3}{4}\right) \div \left(\frac{1}{5}\right)$?

(22) How do you divide a fraction by another fraction? Do you need common denominators in this case?

(23) Perform the following multiplications and simplify your answer:

$8 \div 4$ $\qquad \frac{8}{15} \div \frac{4}{15}$ $\qquad \frac{2}{5} \div \frac{3}{5}$ $\qquad \frac{4}{7} \div \frac{10}{7}$

$\frac{10}{11} \div \frac{7}{12}$ $\qquad \frac{8}{15} \div \frac{1}{5}$ $\qquad \frac{2}{7} \div \frac{3}{5}$ $\qquad \frac{2}{9} \div \frac{1}{4}$

$\frac{7}{12} \div \frac{5}{18}$ $\qquad \frac{1}{2} \times \frac{7}{3} \div \frac{11}{6}$ $\qquad \frac{7}{10} \div \frac{3}{20} \times \frac{2}{3}$ $\qquad \frac{3}{8} \div \frac{7}{12} \div \frac{2}{3}$

(24) If you walk $\frac{1}{4}$-th of a mile from your home to the bus-station, $\frac{2}{3}$-rd of a mile from the bus-station to the train station, and $\frac{3}{5}$-th of a mile from the train station to the college, how much distance have you walked?

(25) If the length of rectangle is $\frac{2}{5}$-th of a foot and the width is $\frac{2}{7}$-th of a foot, then find the perimeter of the rectangle.

2. Real numbers

(1) What are the natural numbers?

(2) What are the whole numbers? Give a whole number which is not a natural number.

(3) What are signed numbers? Give five examples of positive numbers and five examples of negative numbers.

(4) What is the sign for zero?

(5) What are the integers? Give five examples of integers which are not whole numbers.

(6) What are the rational numbers? Give five examples of rational numbers which are not integers.

(7) What are the irrational numbers? Give five examples.

(8) What are the real numbers?

(9) What is the opposite of a number? Give five examples.

(10) What is the absolute value of a number? Give five examples.

(11) How do we compare numbers?

(12) Write in words what the following mean:

(a) $<$
(b) $>$
(c) $\leq$
(d) $\geq$
(e) $x < 4$
(f) $-10 > -12$
(g) $0.9 < 1.1$
(h) $x \geq 3$

(13) State true or false:

(a) $3 < 10$
(b) $3 \leq 10$
(c) $-2 > -10$
(d) $10 \leq 10$
(e) $10 \geq 10$
(f) $|-10| = (-(-(-(-(10)))))$

(14) Fill in the blanks:

(a) The smallest natural number is ____________.
(b) The largest natural number is ____________.
(c) The smallest whole number is ____________.
(d) The largest whole number is ____________.

(e) The smallest integer is ____________.

(f) The smallest positive integer is ____________.

(g) The largest negative integer is ____________.

(h) -3____________ -10.

(i) 0____________ -2.

(j) 10____________10.

(15) Plot the following numbers on the given number line:
0, -1, -2, -3, -4, 1,2,3,4, 3.5, 0.75, -1.25, $1\frac{1}{4}$, $-2\frac{1}{3}$, $-3\frac{2}{5}$.

(16) Simplify:

- $-(-3)$
- $|-(-(-(3)))|$
- $(-(-(-(-(-(-3))))))$

3. Adding and subtracting real numbers

(1) Explain as though you are a teacher why $3+2=5$ (say, using 3 chocolates and 2 chocolates).

(2) Explain using the number line why $3+2=5$.

(3) Explain in your words why $10+(-4)=6$.

(4) Explain using the number line why $10+(-4)=6$.

(5) Explain using the number line what $-4+(-5)$ is.

(6) Explain using the number line what $-7+6$ is. How about $6+(-7)$?

(7) Write in your own words, how you would add two numbers of the same sign. Give five examples.

(8) Write in your own words, how you would add two numbers of different signs. Give five examples.

(9) State the commutative property of additionn Give 5 examples.

(10) State the associative property of addition. Give 5 examples.

(11) Does the commutative property hold for subtraction? Explain using an example.

(12) Does the associative property hold for subtraction? Explain using an example.

(13) What is the additive inverse of a number?

(14) What is the additive identity property of 0? Give 5 examples.

(15) Find:

(a) $12 + 13$
(b) $12 - 13$
(c) $-12 - 13$
(d) $-12 + 13$
(e) $(34) + (9)$
(f) $(-34) + (-9)$
(g) $(-34) - (9)$
(h) $(34) + (-9)$
(i) $\left(\frac{3}{4}\right) + \left(\frac{5}{6}\right)$

(j) $\left(-\frac{3}{8}\right) - \left(-\frac{5}{12}\right)$

(k) $\left(-2\frac{1}{6}\right) + \left(3\frac{2}{3}\right)$

(l) $\left(-2\frac{1}{6}\right) - \left(3\frac{1}{2}\right)$

(m) $\left(-5\frac{1}{8}\right) + \left(1\frac{3}{8}\right)$

(n) $-25.334 + 22.112$

(o) $1.392 - 0.887$

(p) $6.703 - 5.434$

(q) $25.67 - 98.10$

(r) $12 - (-8)$
(s) $-12 - (-8)$

(t) $-23-(-52)$

(u) $-3.567-(-4.123)$

(v) $-24.345-(-43.123)$

(w) $23.543-(67.345)$

(x) $\left(-3\frac{1}{7}\right)-\left(-5\frac{2}{7}\right)$

(y) $\left(1\frac{1}{8}\right)-\left(-3\frac{2}{8}\right)$

(16) Subtract (-13) from 24.

(17) Subtract (-3.45) from -0.75.

(18) Subtract $-3\frac{1}{4}$ from $-\frac{3}{8}$.

(19) Find the difference of -0.35 and -4.57.

(20) Find the difference of $-1\frac{2}{3}$ and $2\frac{5}{6}$.

(21) The temperature in Potsdam rose from -5^oF to 28^oF. What was the rise in temperature?

(22) Water rushed from the depth of 3 feet below sea-level to a height of 12 feet above sea-level. What was the rise in water-level?

4. Multiplying and dividing real numbers

(1) $30 \div 5$ can be rewritten as $5\overline{)30}$. Here 30 is the **dividend** and 5 is the **divisor**. What is the **quotient** in this case?

(2) Identify the divisor, dividend and quotient in each of the cases:

  (a) $48 \div 8 = 6$. Here, dividend = ______, divisor = ______, and quotient = ______.

  (b) $32 \div 4 = 8$. Here, dividend = ______, divisor = ______, and quotient = ______.

  (c) $a \div b = c$. Here, dividend = ______, divisor = ______, and quotient = ______.

(3) Fill in the blanks:

  (a) $+ \div + =$ ____________

  (b) $+ \div - =$ ____________

  (c) $- \div + =$ ____________

  (d) $- \div - =$ ____________

(4) What is the meaning of 15 divided by three is 5?

(5) What is the meaning of -15 divided by three is -5? Do you now see $- \div + = -$?

(6) What is $(-3) \div 1$?

(7) What is $(-3) \div (-1)$?

(8) State the commutative property of multiplication. Give 5 examples.

(9) State the associative property of multiplication. Give 5 examples.

(10) Does the commutative property hold for division? Explain using an example.

(11) Does the associative property hold for division? Explain using an example.

(12) Give 5 examples of multiplications involving 0.

(13) Give 5 examples of divisions involving 0. What should one be careful about in this situation?

(14) Find:

(a) $14 \times (-2)$

(b) $(-14) \times (-7)$

(c) $(-14) \times (-2)$

(d) $24 \times (-8)$

(e) $(-44) \times (-9)$

(f) $(-11) \times (0)$

(g) $(0) \times (-11)$

(h) $\frac{3}{4} \times \frac{1}{3}$

(i) $\left(-\frac{4}{5}\right) \times \left(\frac{3}{4}\right)$

(j) $\left(-\frac{11}{13}\right) \times \left(-\frac{26}{33}\right)$

(k) $\left(-3\frac{1}{7}\right) \times \left(-2\frac{1}{3}\right)$

(l) $\left(-4\frac{3}{5}\right) \times \left(5\frac{5}{7}\right)$

(m) $14 \div (-2)$
(n) $(-14) \div (-7)$
(o) $(-14) \div (-2)$
(p) $24 \div (-8)$
(q) $(-44) \div (-9)$
(r) $(-11) \div (0)$
(s) $(0) \div (-11)$
(t) $\frac{3}{4} \div \frac{1}{3}$ (**Remember:** Division by a fraction is multiplication by its reciprocal).

(u) $\left(-\frac{4}{5}\right) \div \left(\frac{3}{4}\right)$

(v) $\left(-\frac{15}{34}\right) \div \left(-\frac{5}{17}\right)$

(w) $\left(-3\frac{1}{7}\right) \div \left(-2\frac{1}{3}\right)$

(x) $\left(-4\frac{3}{5}\right) \div \left(5\frac{1}{2}\right)$

(y) 2×0

(z) $3 - 0$

(a) 0×3

(b) $0 - 2$

(c) $2 \div 0$

(d) $0 \div 2$

(e) $0 \div 0$

(f) $\dfrac{2-3}{0-3}$

(g) $0 - (-(2))$

(h) $\dfrac{5+(-5)}{-2+2}$

(i) $\dfrac{12+(-5)}{-8+2}$

(j) $\dfrac{\left(\dfrac{2}{3}\right)}{12}$

(k) $\dfrac{12}{\left(\dfrac{2}{3}\right)}$

(l) $\dfrac{\left(\dfrac{2}{3} - \dfrac{1}{5}\right)}{\left(\dfrac{1}{3} + \dfrac{2}{5}\right)}$

(a) $$\frac{\left(\frac{2}{3} \times \frac{1}{5}\right)}{\left(\frac{1}{3} \div \frac{2}{5}\right)}$$

(b) $$\frac{\left(\frac{2}{3} - \frac{1}{5} + \frac{1}{2}\right)}{\left(\frac{1}{3} + \frac{2}{5} - \frac{1}{2}\right)}$$

(c) $$\frac{\left(\frac{1}{3} - \frac{3}{5} + \frac{1}{2}\right)}{3}$$

(d) $$\frac{4}{\left(\frac{1}{3} - \frac{3}{5} + \frac{1}{2}\right)}$$

5. Exponents and order of operations

(1) In the expression 2^5, the number 2 is the **base** and the number 5 is the **exponent**. Identify the base and the exponent in the following expressions:
   (a) 3^4. Here the base is ______ and the exponent is ______.
   (b) $(-3)^4$. Here the base is ______ and the exponent is ______.
   (c) -3^4. Here the base is ______ and the exponent is ______.
   (d) $\left(\frac{2}{3}\right)^3$. Here the base is ______ and the exponent is ______.

(2) Evaluate the following:
   (a) 2^4
   (b) $(-2)^4$
   (c) -2^4
   (d) 2^5
   (e) -2^5
   (f) 2^0
   (g) 0^0
   (h) 0^2
   (i) $-(-4^2)$
   (j) $-(-4)^2$
   (k) $\left(\frac{3}{4}\right)^3$
   (l) $\left(\frac{-3}{4}\right)^3$
   (m) $\left(\frac{3}{-4}\right)^3$

(3) Complete the following table of squares:

$0^2 =$	$6^2 =$	$12^2 =$	$18^2 =$
$1^2 =$	$7^2 =$	$13^2 =$	$19^2 =$
$2^2 =$	$8^2 =$	$14^2 =$	$20^2 =$
$3^2 =$	$9^2 =$	$15^2 =$	$30^2 =$
$4^2 =$	$10^2 =$	$16^2 =$	$40^2 =$
$5^2 =$	$11^2 =$	$17^2 =$	$50^2 =$

(4) Complete the following table of cubes:

$0^3 =$	$3^3 =$	$6^3 =$	$9^3 =$
$1^3 =$	$4^3 =$	$7^3 =$	$10^3 =$
$2^3 =$	$5^3 =$	$8^3 =$	$100^3 =$

(5) Write down the **order of operations**.

(6) Evaluate:

(a) $2-3-4$

(b) $2-(3-4)$

(c) $12 \div 4 \div 2$

(d) $12 \div (4 \div 2)$

(e) $(-2)^3 - \sqrt{225} + 81 \div 3 + 6$

(f) $3(12-4)^2 - \sqrt{48 \div 3}$

(g) $\left(-\dfrac{3}{2}\right)^3 + \left(3\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right)^2$

(h) $-4[12-2(-7)]$

(i) $2^3 - 5\{6^2 - \sqrt{3^2+4^2} - (-(-20))\}$

(j) $\dfrac{(-2)^2+5}{3(12-7)}$

(k) $(3-7-8)-\sqrt{36+64}$

(l) $2(3+2)^2-[4-(2+7\{3-9\})]$

(m) $\dfrac{7^2-5^2}{7-5}$

(n) $\dfrac{3^2-4^2}{11}+\dfrac{(-3)^2+4^2}{11}$

(o) $\left(\dfrac{3}{2}\right)^2-2\left(\dfrac{3}{2}\right)\left(\dfrac{1}{2}\right)+\left(\dfrac{1}{2}\right)^2$

(p) $\left(\dfrac{3}{2}-\dfrac{1}{2}\right)^2$

(q) $\left(\dfrac{3}{2}\right)^2-\left(\dfrac{1}{2}\right)^2$

6. Evaluating algebraic expressions

Evaluate the expressions for the given values of variables.

(1) $x^2 - 3x + 4$

(a) $x = 0$

(b) $x = 2$

(c) $x = -2$

(2) $\left(\dfrac{2a - b}{3a - b^2 + 4}\right)$

(a) $a = 0, b = 0$

(b) $a = -2, b = 3$

(c) $a = -2, b = -3$

(3) $\sqrt{\dfrac{x^2 + y^2}{x^2 y^2}}$

(a) $x = 1, y = -1$

(b) $x = -2, y = 3$

(c) $x = -2, y = -3$

(4) $2[-x-y]^2+3(y-x)$

(a) $x=0, y=-3$

(b) $x=-1, y=-2$

(c) $x=4, y=-1$

(5) $\dfrac{3x+y}{4xy}$

(a) $x=0, y=-3$

(b) $x=-1, y=-2$

(c) $x=4, y=-1$

(6) $a^3-3a^2b+3ab^2-b^3$

(a) $a=2, b=3$

(b) $a=0, b=-2$

(c) $a=-1, b=-3$

(7) $(a-b)^3$

(a) $a=2, b=3$

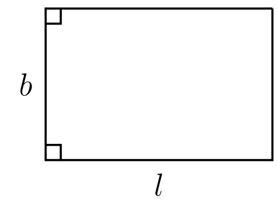
(b) $a=0, b=-2$

(c) $a=-1, b=-3$

Evaluate the following formulae for the given values of variables:

(1) Area of a rectangle formula: $A = lb$

(a) $l = 3, b = 4$

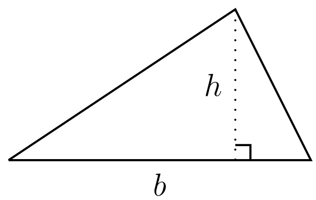


(b) $l = 12, b = 22$

(c) $l = 30, b = 20$

(2) Area of a triangle formula: $A = \dfrac{bh}{2}$

(a) $b = 3, h = 10$

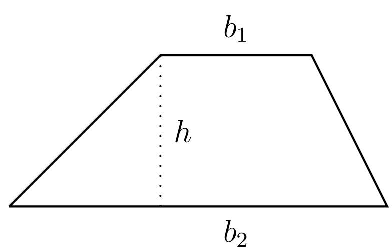


(b) $b = 22, h = 13$

(c) $b = 7, h = 9$

(3) Area of a trapezoid formula: $A = \dfrac{(b_1 + b_2)h}{2}$

(a) $b_1 = 2, b_2 = 3, h = 5$

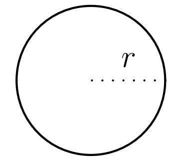


(b) $b_1 = 4, b_2 = 12, h = 10$

(c) $b_1 = 12, b_2 = 14, h = 8$

(4) Area of a circle formula: $A = \pi r^2$ (**Don't let π trouble you. Leave it in your answer.**)

(a) $r = 3$

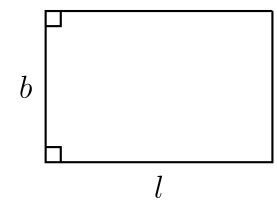


(b) $r = \frac{3}{2}$

(c) $r = \sqrt{5}$

(5) Perimeter of a rectangle formula: $P = 2l + 2b$

(a) $l = 3, b = 23$

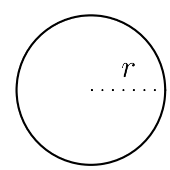


(b) $l = 14, b = 15$

(c) $l = 22, b = 29$

(6) Circumference of a circle formula: $C = 2\pi r$

(a) $r = 3$

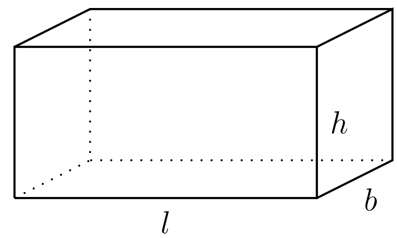


(b) $r = \frac{3}{2}$

(c) $r = \sqrt{5}$

(7) Volume of a rectangular box formula: $V = lbh$

(a) $l = 2, b = 3, h = 5$

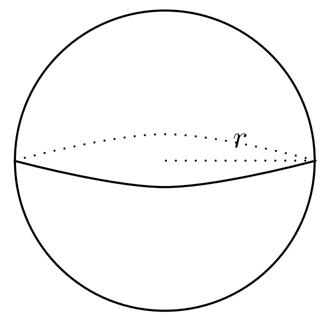


(b) $l = 4, b = 12, h = 10$

(c) $l = 12, b = 14, h = 8$

(8) Volume of a sphere formula: $V = \dfrac{4}{3}\pi r^3$

(a) $r = 3$



(b) $r = \dfrac{3}{2}$

(c) $r = \sqrt{5}$

(9) $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

(a) $x_1 = -1, x_2 = -3, y_1 = 5, y_2 = 4$

(b) $x_1 = -100, x_2 = -100, y_1 = 5, y_2 = 4$

(c) $x_1 = -1, x_2 = -3, y_1 = 4, y_2 = 4$

7. Transition to Algebra

Translate each phrase in English to an expression in mathematics. If you introduce variables, then state in **complete sentences** what they stand for.

(1) Twenty plus x

(2) The sum of a and 13

(3) The difference between a and 13

(4) Five less than b

(5) The sum of 32 and two times x

(6) Three less than four times m

(7) Twice the sum of m and 3

(8) The quotient of the sum of x and 4 and twice y

(9) My salary plus \$25

(10) Eight less than twice Rob's age

(11) Subtract an unknown number from 5

(12) Subtract 5 from an unknown number

(13) The square of the result when 10 is subtracted from twice a number

(14) The sum of two unknown numbers

(15) The sum of two unknown consecutive integers

(16) The sum of two unknown consecutive odd integers

(17) The sum of two unknown consecutive even integers

Translate each sentence in English to a mathematical statement. If you introduce variables, then state in **complete sentences** what they stand for.

(1) The product of two unknown consecutive odd integers is 143.

(2) The area of a rectangle is 40 sq. ft.

(3) Rob's age is 10 less than twice Tom's age.

(4) The sum of the squares of two numbers is 32.

(5) When 6 is subtracted from 5 times a number, the result is 33.

(6) The width is 7 less than twice the length of the rectangle.

(7) The height is the square of the sum of three times the base and 4.

(8) One number is 4 times the sum of another number and 8.

(9) The product of two numbers is the square of the sum of the two numbers.

(10) One-fourth the product of two numbers is a third of the square of the sum of the two numbers.

(11) The product of a number and a quarter of another is the square of the sum of a fifth of the first number and a sixth of the second number.

(12) Three-quarters the product of two numbers is four-fifths the sum of the two numbers.

(13) Seven-tenths the sum of two numbers is the one-half their difference.

(14) Five-elevenths the square of the sum of two numbers is the cube of the first number.

(15) Two-thirds the difference of two numbers is three-fourths the sum of the two numbers plus five.

8. Solving linear equations

Here we work with equations. An **equation** is a mathematical statement involving equality (=). A solution of an equation is a number that when substituted for the variable gives a true statement.

For example, $x = 3$ **solves** the equations $2x = 6$ and $x - 5 = -2$.

(1) Is -4 a solution of $2x = 6$?

(2) Is -3 a solution of $x = 3$?

(3) Is 1 a solution of $5x = 0$?

(4) Is -5 a solution of $-3x = -15$?

(5) Write 5 equations which are solved by -4.

(6) Write 5 equations which are solved by -5.

(7) Write 5 equations which are solved by 2.

Think of an $=$ sign as a **balance**. Whatever you do to one side, you must do to the other side as well. So,

- The **same** number may be **added** to both sides.
- The **same** number may be **subtracted** from both sides.
- Both sides may be **multiplied** by the **same** number.
- Both sides may be **divided** by the **same** non-zero number.

Solve and check the equations.

(1) $x + 3 = 11$

(2) $12 = x - 4$

(3) $-18 + x = 20$

(4) $2x = 10$

(5) $-20 = \dfrac{x}{5}$

(6) $-30 = -6x$

(7) $\dfrac{x}{2} = 10$

(8) $2x + 3 = 11$

(9) $12 = 3x - 4$

(10) $-18 + 4x = 20$

(11) $2x - 12 = 10$

(12) $-20 = \frac{x}{5} + \frac{4}{3}$

(13) $-30 = -6x + 7$

(14) $\frac{x}{2} - \frac{2}{3} = 10$

(15) $2x + 3 = 5x + 11$

(16) $12 - 2x = 3x - 4$

(17) $-18 + 4x = 7x + 20$

(18) $2x - 12 = 10 - x$

(19) $-20 + \frac{2x}{3} = \frac{x}{5} + \frac{4}{3}$

(20) $4x - 30 = -6x + 7$

(21) $\frac{x-4}{2} - \frac{2}{3} = 10$

(22) $2(x + 3) = 5x + 11$

(23) $12 - 2x = 3(x - 4) + 5(x - 10)$

(24) $-18 - 4(x - 3) = 7(x + 5) + 20$

(25) $-2(x + 5) - 12 = 3(10 - x)$

(26) $-20 + \dfrac{2x}{x+3} = \dfrac{4}{3}$

(27) $4(x+11) - 30 = -6(x-5) + 7(x-3)$

(28) $\dfrac{2x-3}{2} - \dfrac{2}{3} = 10x$

Solve the following word-problems. If you introduce variables, then state in **complete sentences** what they stand for.

(1) Twenty plus x is 34. What is x?

(2) The sum of a and 13 is 32. What is a?

(3) Five less than b is 99. What is b?

(4) The sum of 32 and two times x is 45. Find x.

(5) Three less than four times m is 100. Find m.

(6) Twice the sum of m and 3 is 55. Find m.

(7) The quotient of the sum of x and 4 and 10 is 4. What is x?

(8) His salary plus \$250 is \$2000. What is his salary?

(9) Eight years less than twice Rob's age is 32 years. What is Rob's age?

(10) When a number is subtracted from 5, the result is 32. Find the number.

(11) When 20 is subtracted from the product of 5 and a number, the result is 12. Find the number.

(12) The square of the result when 10 is subtracted from twice a number is 25. Find the number.

(13) The sum of two consecutive integers is 177. Find the smaller number.

(14) The sum of two consecutive odd integers is 172. Find the larger integer.

(15) The sum of two consecutive even integers is 186. Find both the integers.

(16) The area of a square is 40 sq. ft. Find the length of its side.

(17) Rob's age is 10 less than twice Tom's age. If Rob's age is 30 years, then what is Tom's age?

(18) The sum of the squares of two numbers is 32. If one number is 3, then find the other.

(19) When 6 is subtracted from 5 times a number, the result is 33. Find the number.

(20) The width of a rectangle is 7 less than twice its length. If the perimeter of the rectangle is 18 ft, then what is the length?

(21) The height of a triangle is square of the sum of three times the base and 4. If the height is 49 ft., then what is the base?

(22) One number is 4 times the sum of another number and 8. If the first number is 12, then what is the second number?

(23) Seven-tenths the sum of a number and its third is one-half their difference. Find the number.

(24) Five-elevenths the sum of a number and 5 is two-third the sum of the number and 1. Find the number.

(25) Two-thirds the difference of a number and its fifth is three-fourths the sum of the number plus five. Find the number.

(26) I have some quarters and dimes in my pocket. The number of dimes is twice the sum of the number of quarters and 1. If the total value of the change is \$ 2.00, then find the number of dimes.

(27) I have some dimes, nickels, and quarters in my pocket. The number of nickels is one more than the number of dimes and the number of quarters is one more than the number of nickels. The total value of the change is \$ 1.75. How many quarters do I have?

(28) I have some quarters, dimes, and nickels in my pocket. The number of quarters is the product of 4 and the sum of the number of dimes and 1. The number of nickels is 5 times the difference of the number of dimes and one. If the total value of the change is \$ 4.80, then find the number of quarters, dimes, and nickels.

9. Literal equations

(1) Solve each equation for the specified variable:

(a) $I = Prt$ for t.

(b) $P = 2x + 2y$ for x.

(c) $C = 2\pi r$ for r. (Do not let π worry you. It is just a number).

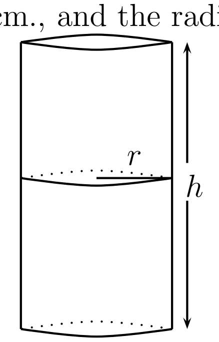
(d) $A = lb$ for b.

(e) $A = P(1 + rt)$ for P.

(f) $M = \dfrac{x + y}{3}$ for x.

(g) $T = \dfrac{2x + 1}{3x - 1}$ for x.

(2) Volume of a right circular cylinder is given by $V = \pi r^2 h$, where r is the radius of the base and h denotes the height of the cylinder. If the volume of a right circular cylinder is 550π cm., and the radius of the base is 15 cm., then find the height.



(3) The final amount A of a deposit of P dollars at a simple rate of interest of r annually, for a period of t years is given by $A = P(1 + rt)$. What was the amount invested 3 years ago at a rate of simple interest .04 if the present amount is \$ 4,000.00?

(4) A certain point (x, y) satisfies the following condition:

$$y = \frac{2x - 1}{3x + 4}$$

If the y-coordinate of the point is 2, then find the x-coordinate of the point.

(5) The slope-intercept form of a line is given by $y = mx + b$ where m is the slope and b the y-intercept of the line. What is the slope of a line which passes through $(1, 2)$ and has y-intercept of 3?

10. Solving linear inequalities

Here we work with inequaties. An **inequality** is a mathematical statement involving inequality symbols ($<, \leq, >$, or $\geq$). A solution set of an inequality is a **set** such that **every** element of the set when substituted for the variable gives a true statement.

For example, $x < 3$ **solves** the equations $2x < 6$ and $x - 5 < -2$. (Check with a few sample numbers less than 3).

(1) Is 4 in the solution set of $2x < 6$?
(2) Is 3 in the solution set of $2x \leq 6$?
(3) Is 3 in the solution set of $2x < 6$?
(4) Write 5 inequalities which are solved by the set of all real numbers less than 4.

The following rules apply while working with inequalities:

- The **same** number may be **added** to both sides.
- The **same** number may be **subtracted** from both sides.
- Both sides may be **multiplied** by the **same positive** number.
- Both sides may be **divided** by the **same positive** number.
- Multiplying or dividing the two sides of an inequality by a **negative** number **changes** the inequality.

(1) Solve the following inequalities and write the solution set in a set-builder notation and interval notation. Further, graph the solution set on a number line.
   (a) $x + 3 < 11$

   (b) $12 > x - 4$

(c) $-18 + x \leq 20$

(d) $-30 \geq -6x$

(e) $\dfrac{x}{-2} \leq 10$

(f) $2x + 3 > 11$

(g) $12 < 3x - 4$

(h) $-18 + 4x \geq 20$

(i) $2x - 12 \leq 10$

(j) $-20 > \dfrac{x}{5} + \dfrac{4}{3}$

(k) $-30 < -6x + 7$

(l) $\frac{x}{2} - \frac{2}{3} \geq 10$

(m) $2x + 3 \leq 5x + 11$

(n) $12 - 2x \geq 3x - 4$

(o) $-18 + 4x < 7x + 20$

(p) $2x - 12 > 10 - x$

(q) $-20 + \frac{2x}{3} < \frac{x}{5} + \frac{4}{3}$

(r) $4x - 30 > -6x + 7$

(s) $\dfrac{x}{2} - \dfrac{2}{3} \leq 10$

(t) $2(x+3) \geq 5x + 11$

(u) $12 - 2x \leq 3(x-4) + 5(x-10)$

(v) $-18 - 4(x-3) \leq 7(x+5) + 20$

(w) $-2(x+5) - 12 > 3(10-x)$

(x) $-20 + \dfrac{2x}{5} < \dfrac{4}{3}$

(y) $4(x+11)-30<-6(x-5)+7(x-3)$

(z) $\dfrac{2x-3}{2}-\dfrac{2}{3}\geq 10x$

(2) If an inequality simplifies to $4>-2$, then what is the solution set?

(3) If an inequality simplifies to $4<-2$, then what is the solution set?

(4) Solve the inequality: $3(x+5)<2(x-1)+x$

(5) Solve the inequality: $3(x+5)>2(x-1)+x$

(6) I aim to receive a grade of at least B in this course. My teacher has informed me that I need an average of at least 80 points in 4 exams to get a grade of B or better. So far, in my three exams I have scored 78, 86, and 60. How much should I score in my final exam to get a grade of B or better?

(7) My financial aid stipulates that my tuition cannot exceed \$ 2,000 this semester. If every course costs me \$380 and the registration fee for a semester is \$ 90, then how what is the maximum number of courses can I register?

(8) The perimeter of a rectangle is to not exceed 28 ft. If the width of the rectangle is 4 feet, then what is the maximum length of the rectangle?

11. Linear equations in two variables

(1) Which of the ordered pairs are solutions to the given points?

(a) $2x + 3y = 6$ $\quad (2, 3), (3, 2), (0, 2), (3, 0), (1, 1).$

(b) $2x + y = 6$ $\quad (2, 3), (3, 2), (0, 2), (3, 0), (1, 1).$

(c) $x + 3y = 6$ $\quad (2, 3), (3, 2), (0, 2), (3, 0), (1, 1).$

(d) $x - 3y = 12$ $\quad (-2, 3), (-3, 2), (0, 12), (3, 0), (1, 1).$

(2) Complete the ordered pairs so that each is a solution to the given equation:

(a) $2x + 3y = 6$ $\quad (1, \quad), (\quad, 2), (\quad, 3), (\quad, 0), (2, \quad).$

(b) $2x + y = 6$ $\quad (\quad, 3), (3, \quad), (\quad, 2), (4, \quad), (0, \quad).$

(c) $x + 3y = 6$ $\quad (2, \quad), (\quad, 2), (0, \quad), (\quad, 0), (\quad, 1).$

(d) $x - 3y = 12$ $\quad (\quad, 3), (-3, \quad), (0, \quad), (\quad, 0), (1, \quad).$

(3) Find five solutions to each of the equations:

(a) $3x + y = 15$

(b) $x + 4y = 5$

(c) $2x - 5y = 10$

(d) $-x - y = 12$

(e) $x + y = -12$.

(f) $y = 3$.

(g) $x = -2$.

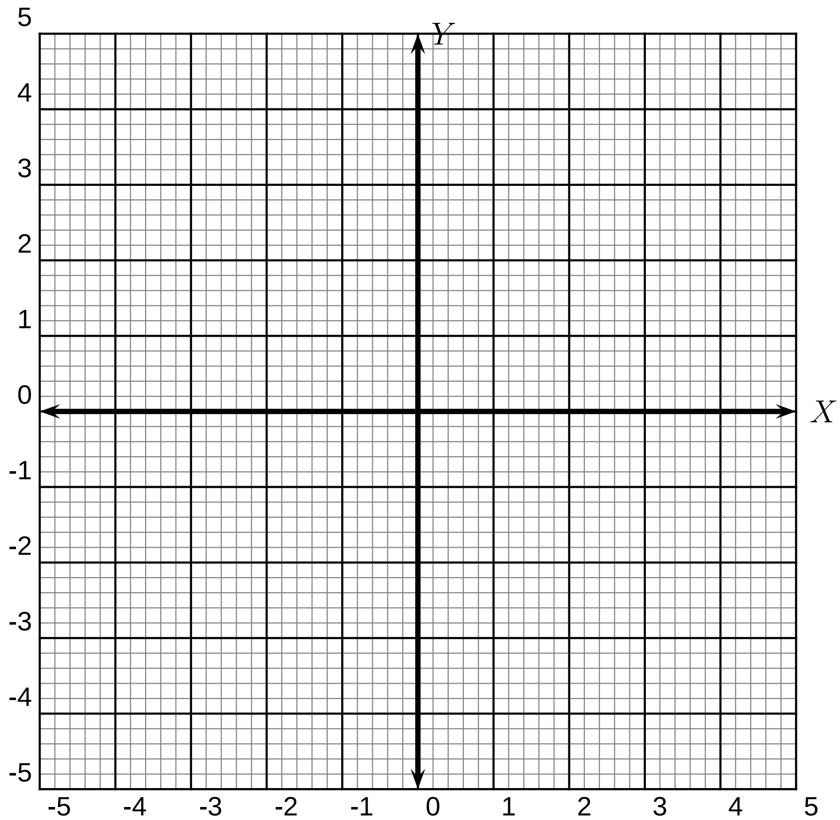
(4) The cost C in dollars of manufacturing x number of televisions is given by the equation $C = 45x + 1200$. Find the cost of manufacturing 1 television, 10 televisions, 20 televisions, and 100 televisions.

(5) The perimeter of a square is given by the formula $P = 4x$ where x denotes the length of a side. What is the perimeter if the length of the side is 4cm, 8cm, 20cm, or 100 cm.

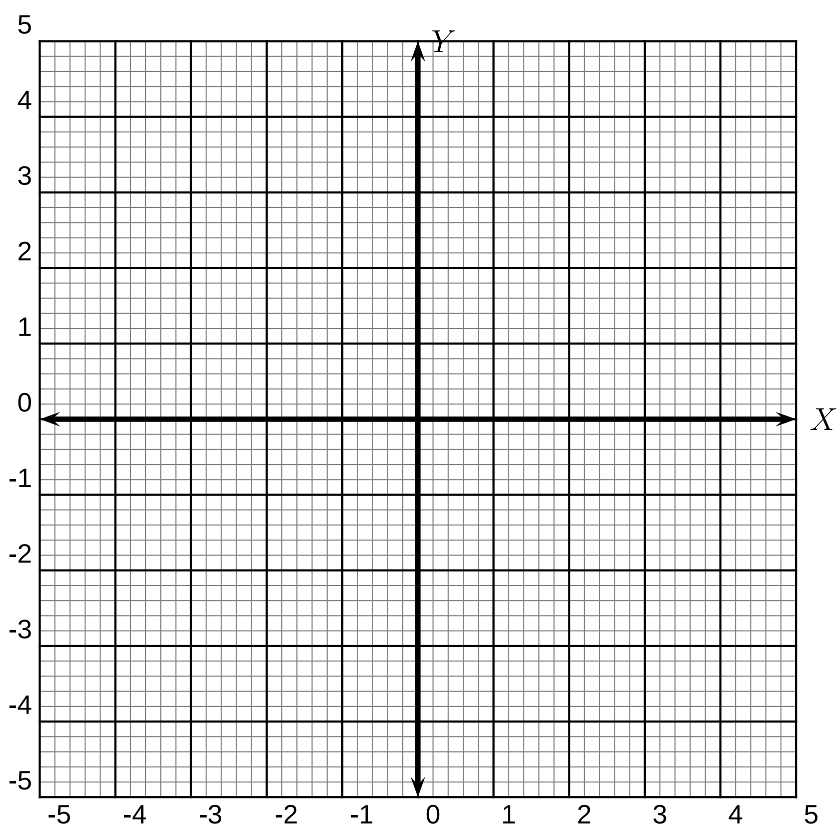
(6) One mile is 1.6 kilometers. In other words, $k = 1.6m$ where m is distance measured in miles, and k is distance measured in kilometers. Find distance in kilometers when the following numbers give distance in miles: 25 miles, 50 miles, 160 miles, 320 miles.

12. The Cartesian coordinate system

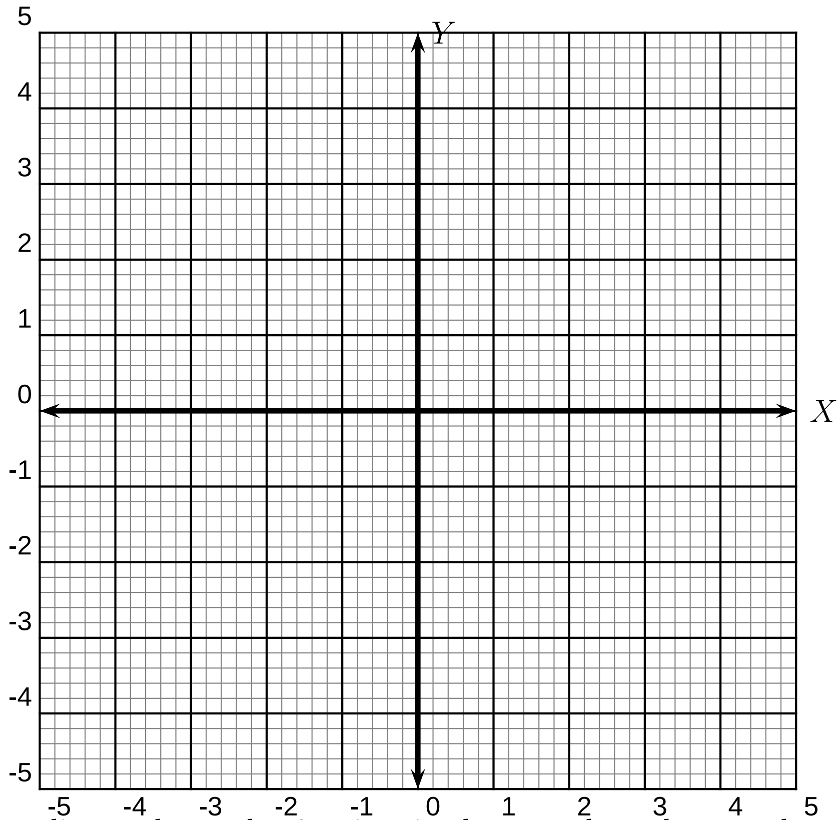
(1) On the given coordinate plane, plot 3 points on the X axis and state the 3 points as ordered pairs.



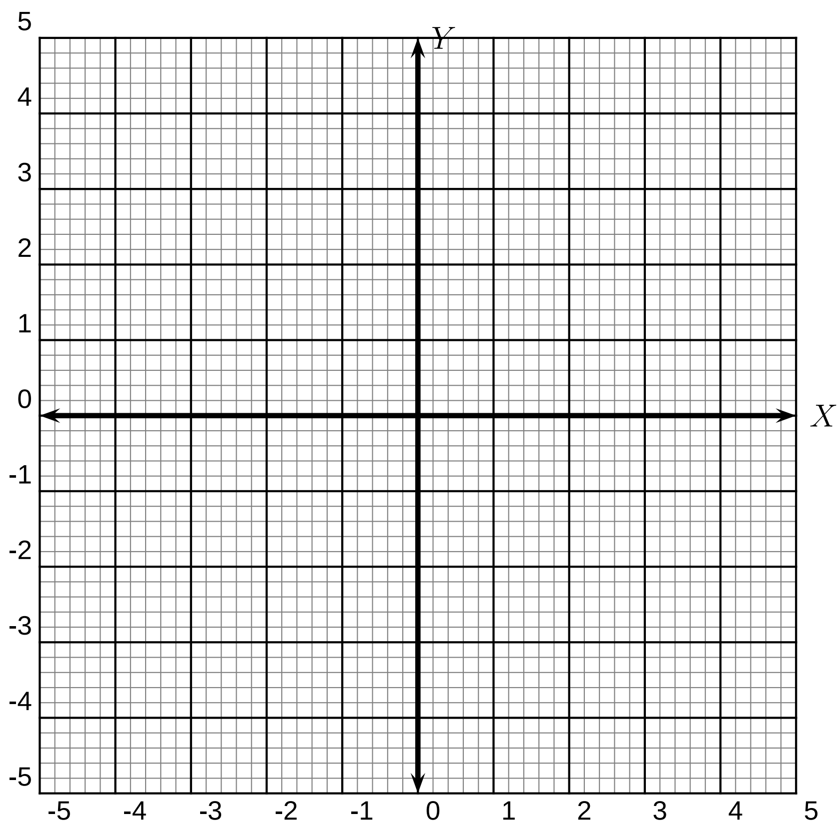
(2) On the given coordinate plane, plot 3 points on the Y axis and state the 3 points as ordered pairs.



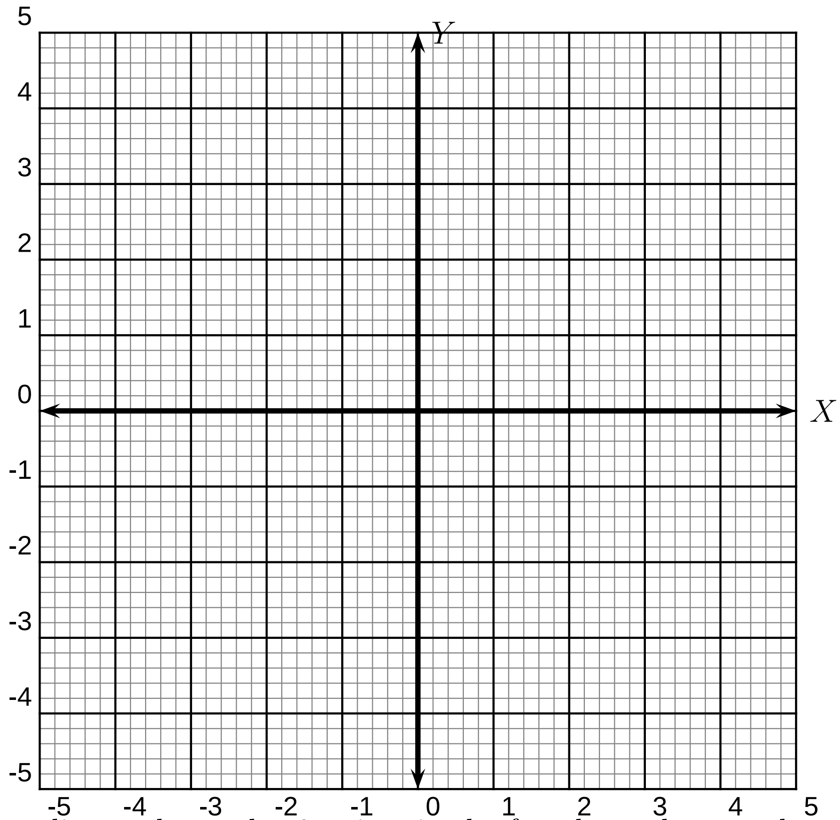
(3) On the given coordinate plane, plot 3 points in the first quadrant and state the 3 points as ordered pairs. Here, are the x coordinates and y coordinates positive or negative?



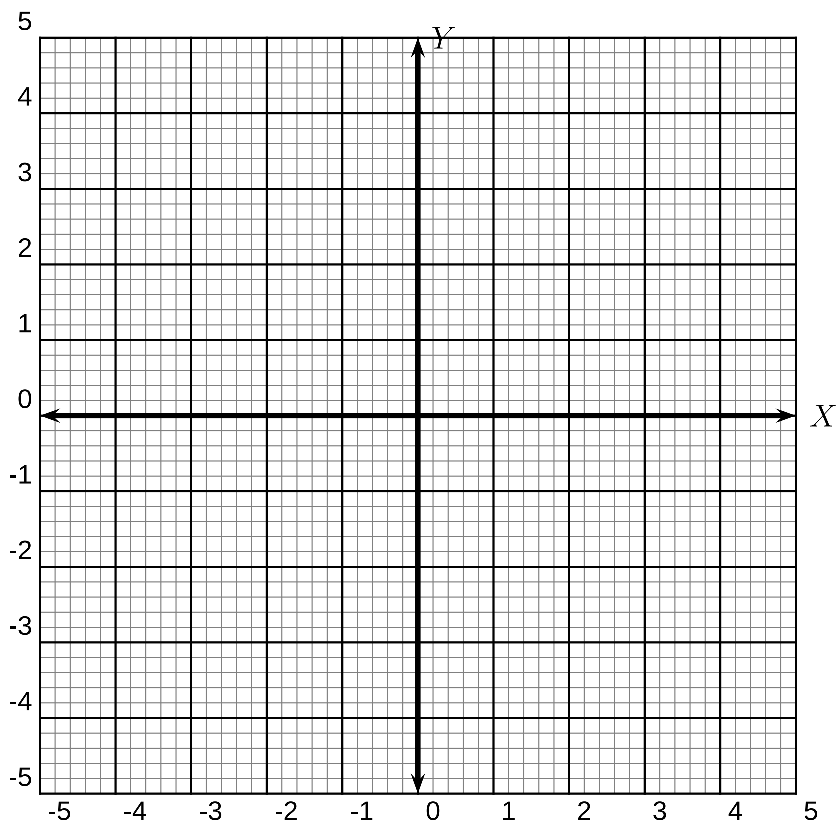
(4) On the given coordinate plane, plot 3 points in the second quadrant and state the 3 points as ordered pairs. Here, are the x coordinates and y coordinates positive or negative?



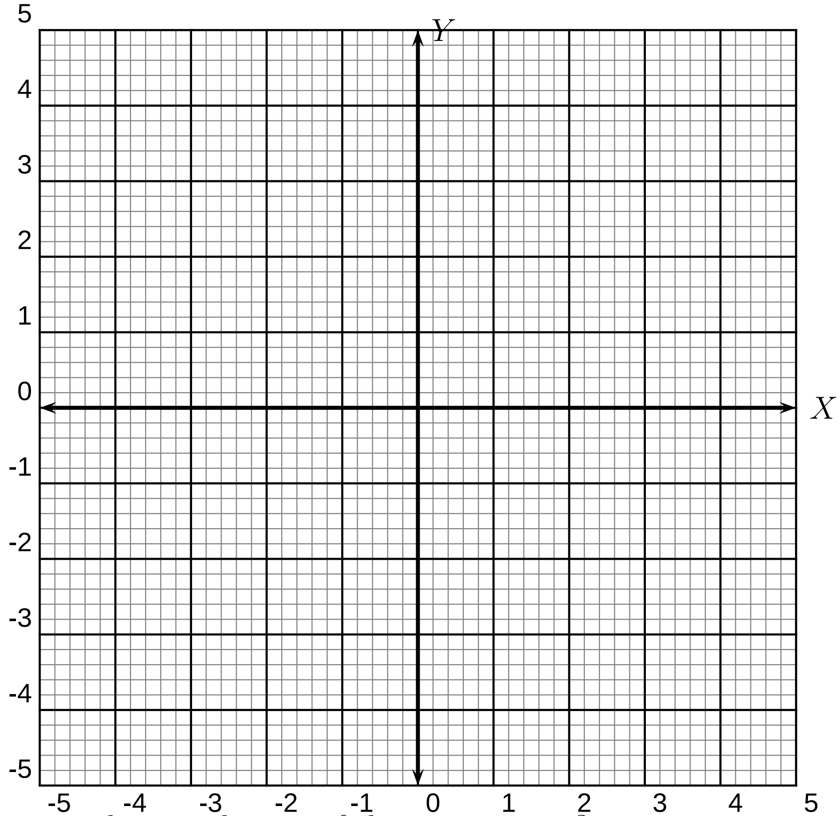
(5) On the given coordinate plane, plot 3 points in the third quadrant and state the 3 points as ordered pairs. Here, are the x coordinates and y coordinates positive or negative?



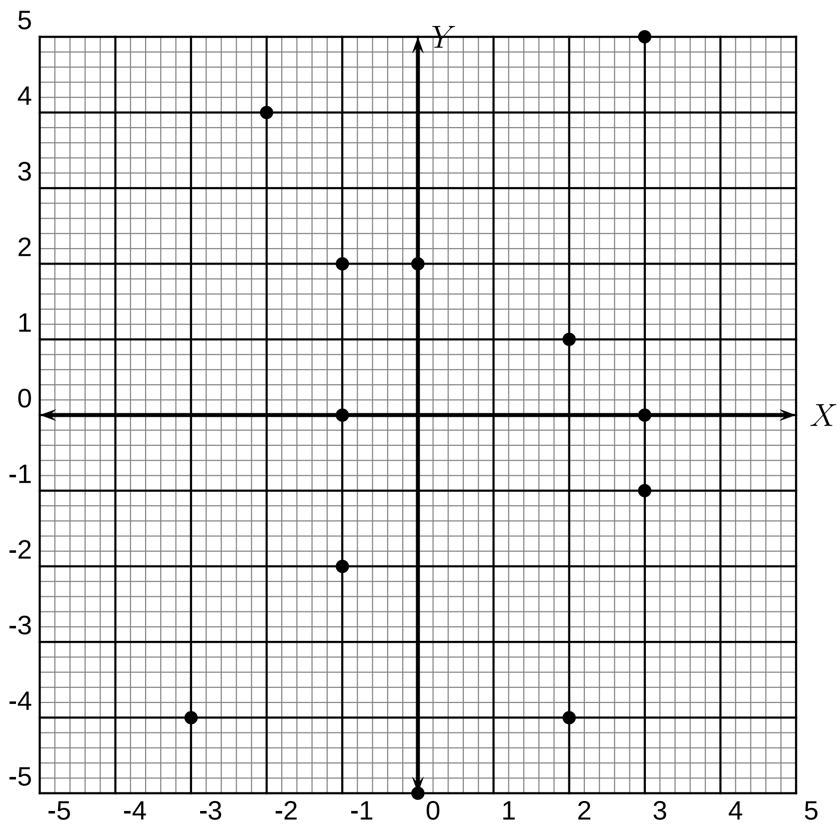
(6) On the given coordinate plane, plot 3 points in the fourth quadrant and state the 3 points as ordered pairs. Here, are the x coordinates and y coordinates positive or negative?



(7) On the given coordinate plane, plot the points $(3, 4)$, $(5, 2)$, $(-2, 3)$, $(-4, 1)$, $(-1, -1)$, $(-5, -2)$, $(2, -3)$, $(1, -4)$, $(3, 0)$, $(0, 5)$, $(-2, 0)$, and $(0, -4)$.



(8) What are the rectangular coordinates of the given points?



(9) The table gives the average temperature t (in degree Centigrades) for the first 6 months, m, of the year. The months are numbered 1 through 6, with 1 corresponding to January. Plot the data given in the table. Name and scale the axes appropriately.

m	1	2	3	4	5	6
t	10	12	14	18	24	30

(10) Plot the baby's weight w (in pounds) recorded at well-baby checkups at the ages x (in months), as described in the table. Name and scale the axes appropriately.

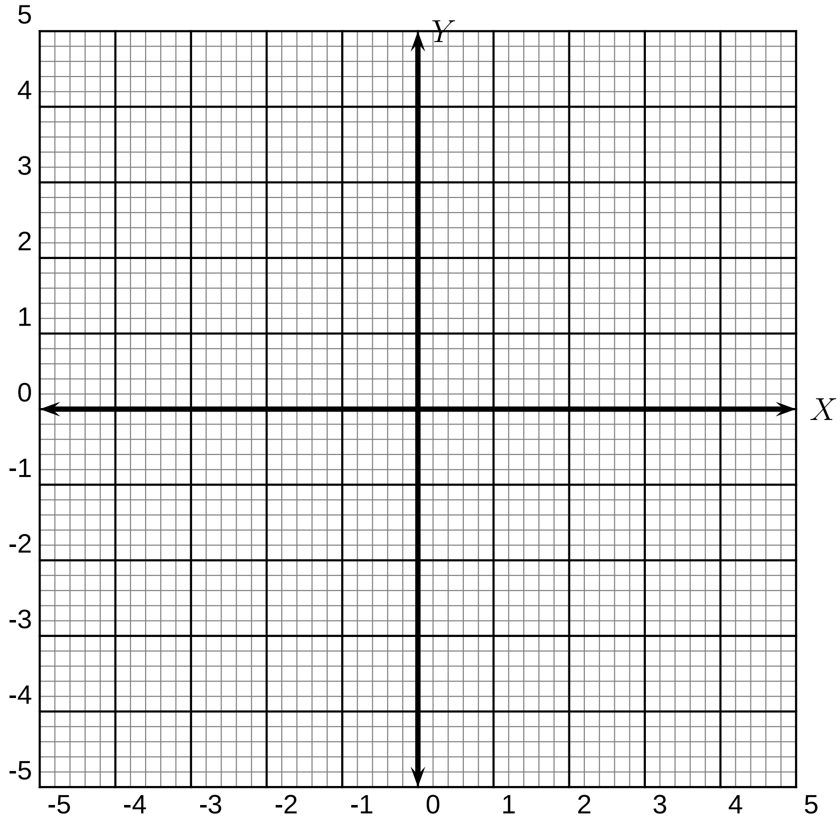
x	0	1	2	6	9	12
w	7	9	13	18	21	25

13. The graph of a linear equation

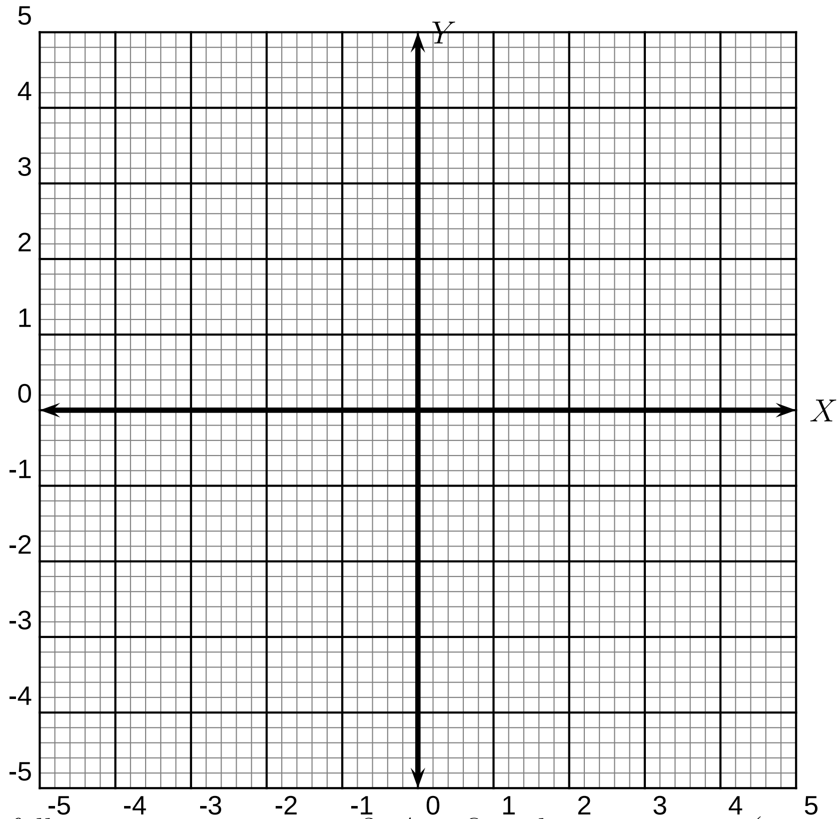
(1) (a) On the given coordinate plane, plot 5 points (x, y) such that $2x+3y=6$. (For example, $(3,0)$ could be one such point).

(b) How many more such points do you think there are?

(c) How do you think all those points are arranged?



(2) (a) On the given coordinate plane, plot 5 points (x, y) such that $2x - 3y = 1$.
(b) How many more such points do you think there are?
(c) How do you think all those points are arranged?

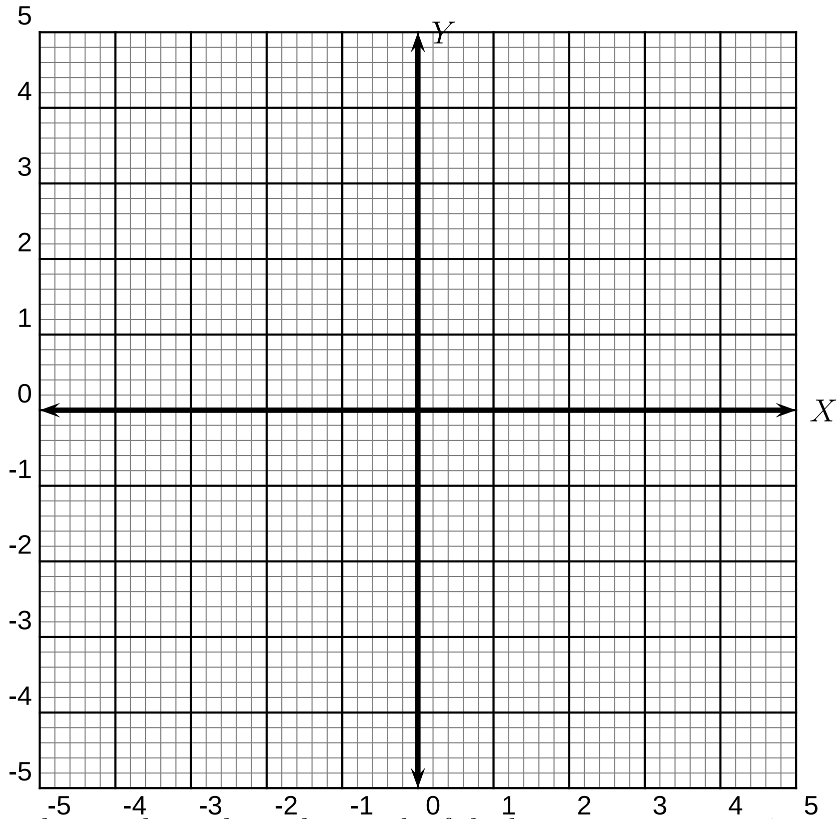


(3) What does the following statement mean? *Any first degree equation (in no more than two variables) has a graph that is a straight line.* Such equations are called **linear**.

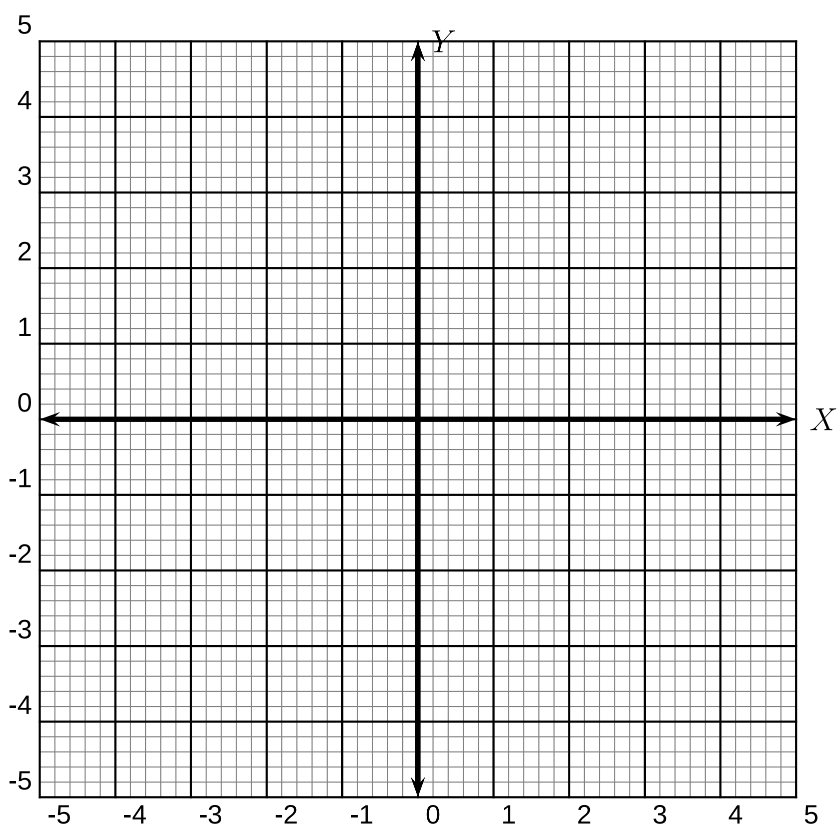
(4) Draw two points on a paper. How many lines passing through both these points can you draw?

Thus, **to draw the graph of a linear equation, we need only two points.**

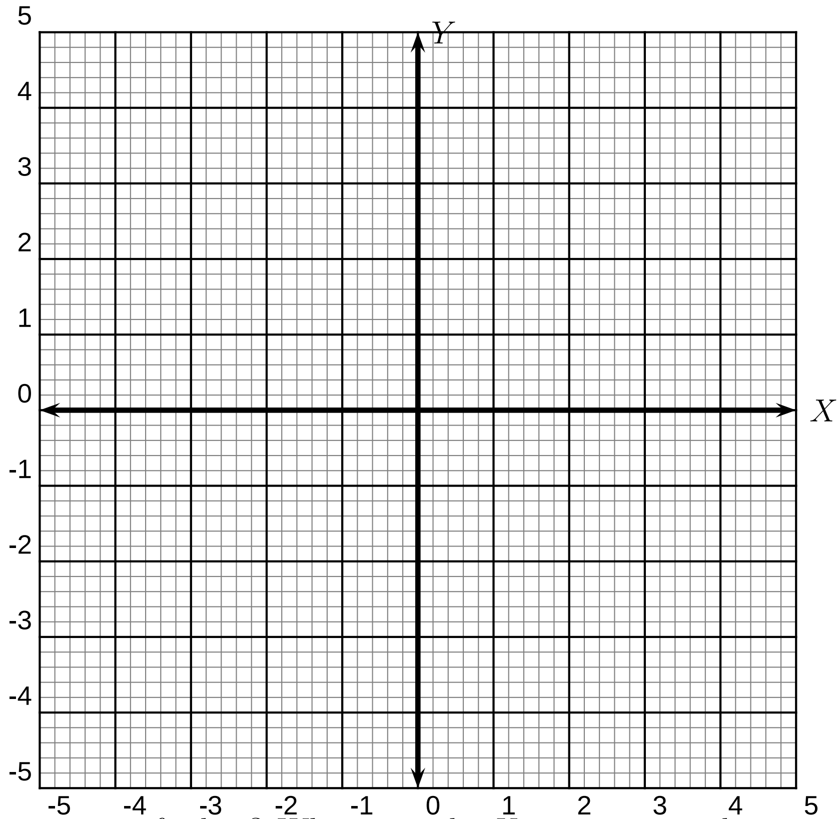
(5) On the given coordinate plane, draw the graph of the linear equation $3x-y=3$. (Remember, all you need are two points to draw a line. There are two easy choices to be made. Which points did you choose? Plot one more point to make sure that your graph is correct.)



(6) On the given coordinate plane, draw the graph of the linear equation $x-4y=8$. (Remember, all you need are two points to draw a line. There are two easy choices to be made. Which points did you choose? Plot one more point to make sure that your graph is correct.)



(7) On the given coordinate plane, draw the graph of the linear equation $-2x - y = 4$. (Remember, all you need are two points to draw a line. There are two easy choices to be made. Which points did you choose? Plot one more point to make sure that your graph is correct.)

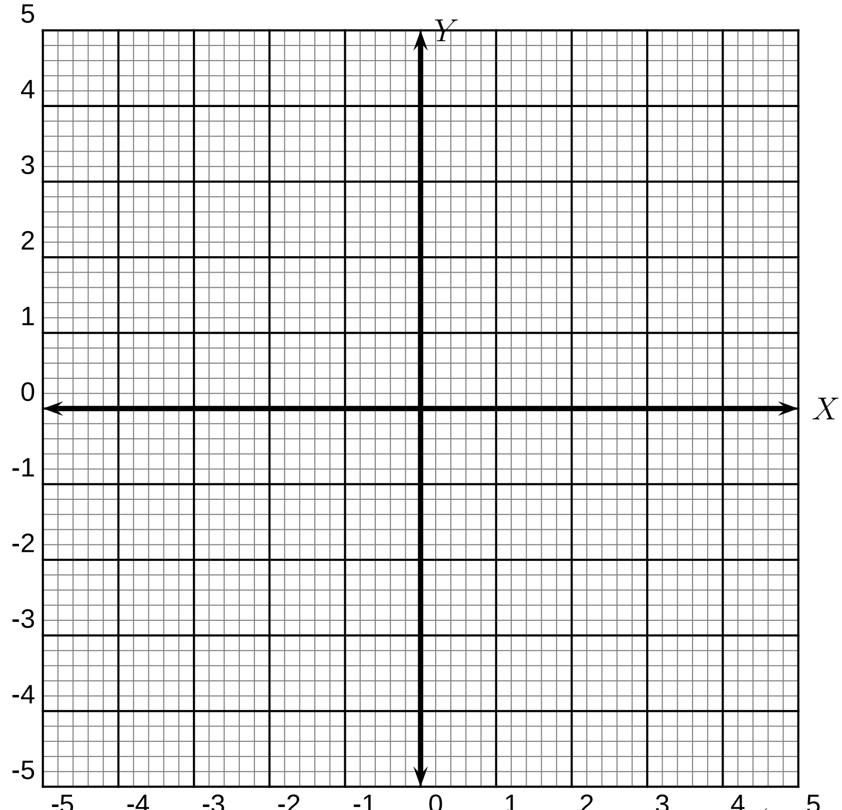


(8) What is the X-intercept of a line? What were the X intercepts in the previous problems?

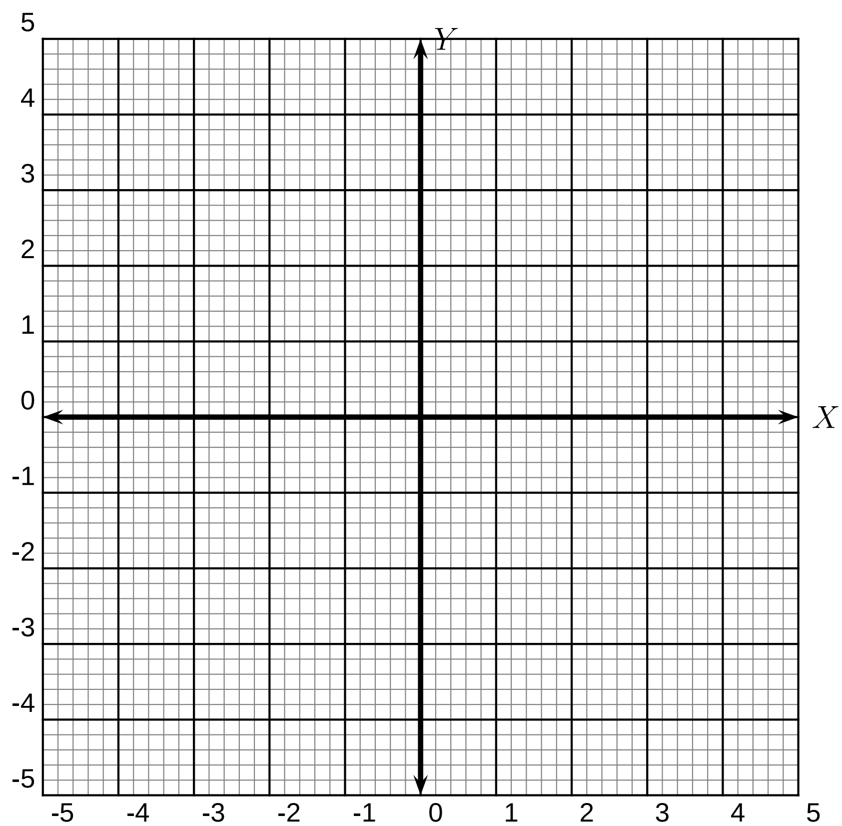
(9) What is the Y-intercept of a line? What were the Y intercepts in the previous problems?

(10) Some lines do not have X or Y intercepts.

(a) On the given coordinate plane, draw the graph of the line $y = 3$. (Which two points did you choose?) What are its X and Y intercepts?



(b) On the given coordinate plane, draw the graph of the line $x = -2$. (Which two points did you choose?) What are its X and Y intercepts?



14. Slope

(1) Imagine a line as a road which is either flat or sloping up or sloping down.

(a) What is the slope of a horizontal line?

(b) Is the slope of a rising line (increasing from left to right) positive or negative?

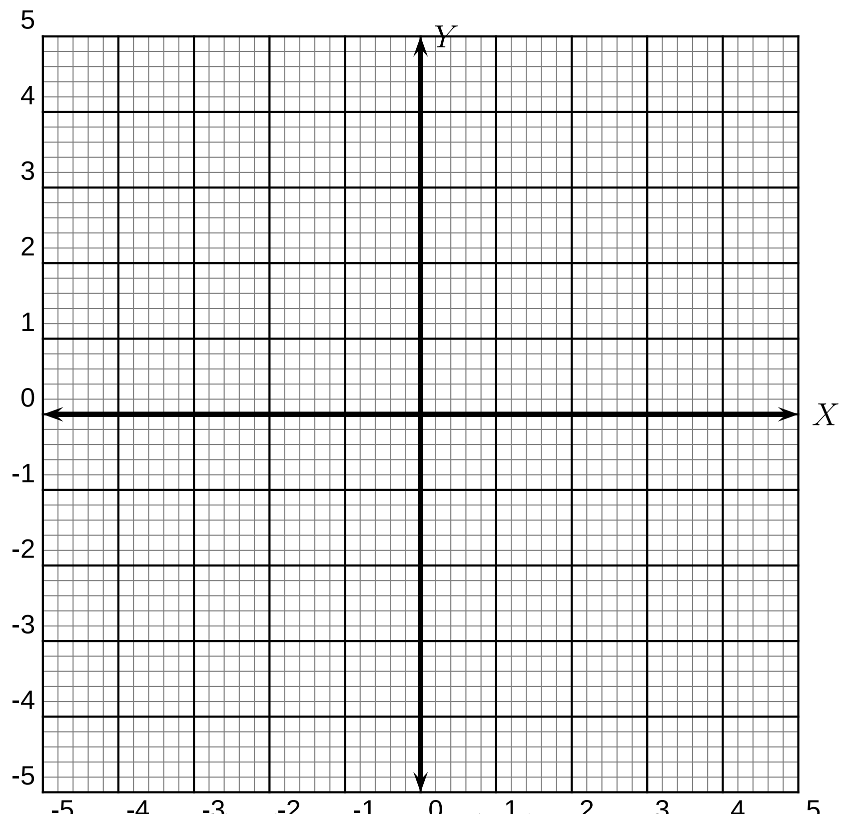
(c) Is the slope of a falling line (decreasing from left to right) positive or negative?

(d) What is the slope of a vertical line?

(2) **Slope** measures the rise of a line with respect to its run. As a formula,

$$slope = \frac{rise}{run}.$$

(a) On the given coordinate plane, draw the line joining $(3, 5)$ and $(2, 3)$.



(b) What is the rise from point $(2, 3)$ to the point $(3, 5)$?

(c) What is the run from point $(2, 3)$ to the point $(3, 5)$?

(d) What is the slope of the line? Would the slope change had you chosen two other points on the same line? (Try and see).

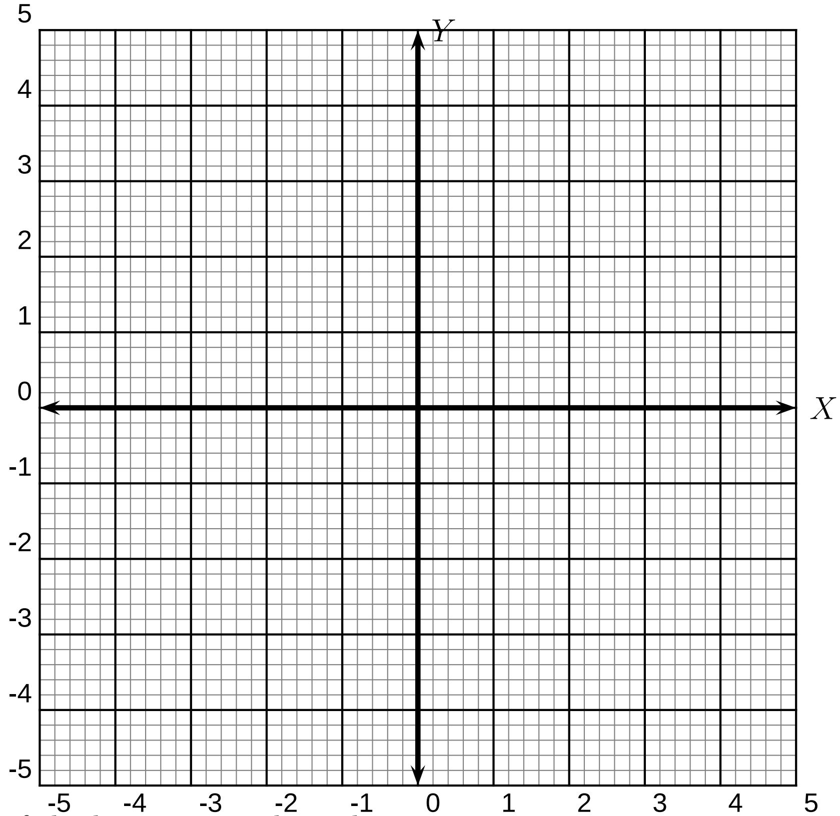
(3) On the given coordinate plane, plot the point $(1, 2)$. Give the rectangular coordinates of the point which is

(a) at a rise of 2 units and a run of 3 units from $(1, 2)$.

(b) at a rise of -2 units and a run of 3 units from $(1, 2)$.

(c) at a rise of 2 units and a run of -3 units from $(1, 2)$.

(d) at a rise of -2 units and a run of -3 units from $(1, 2)$.



(4) Find the slope of the line passing through:

(a) $(1, 1), (-2, 3)$

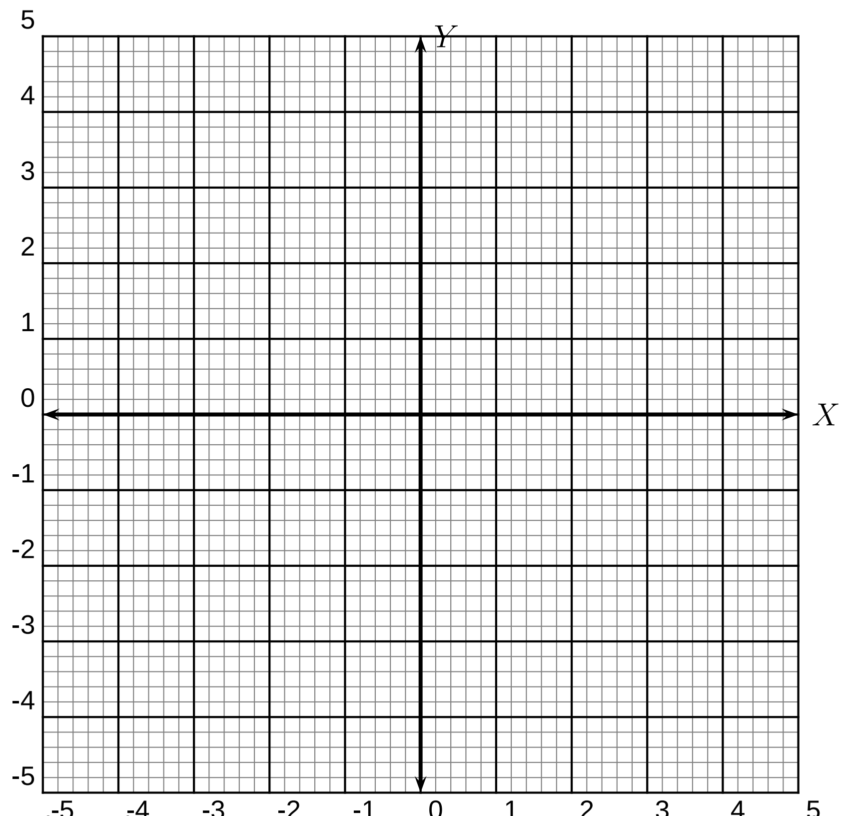
(b) $(1, -1), (4, 3)$

(c) $(-2,1),(5,2)$

(d) $(3,1),(3,3)$

(e) $(4,1),(5,1)$

(f) On the given coordinate plane, draw the lines joining the above pairs of points, and check whether the slopes you found agreed with your intuition?



(5) A linear equation of the form $y = mx + b$ is called the **slope-intercept** form. For this equation, address the following:

(a) When $x = 0$, $y =$ ________

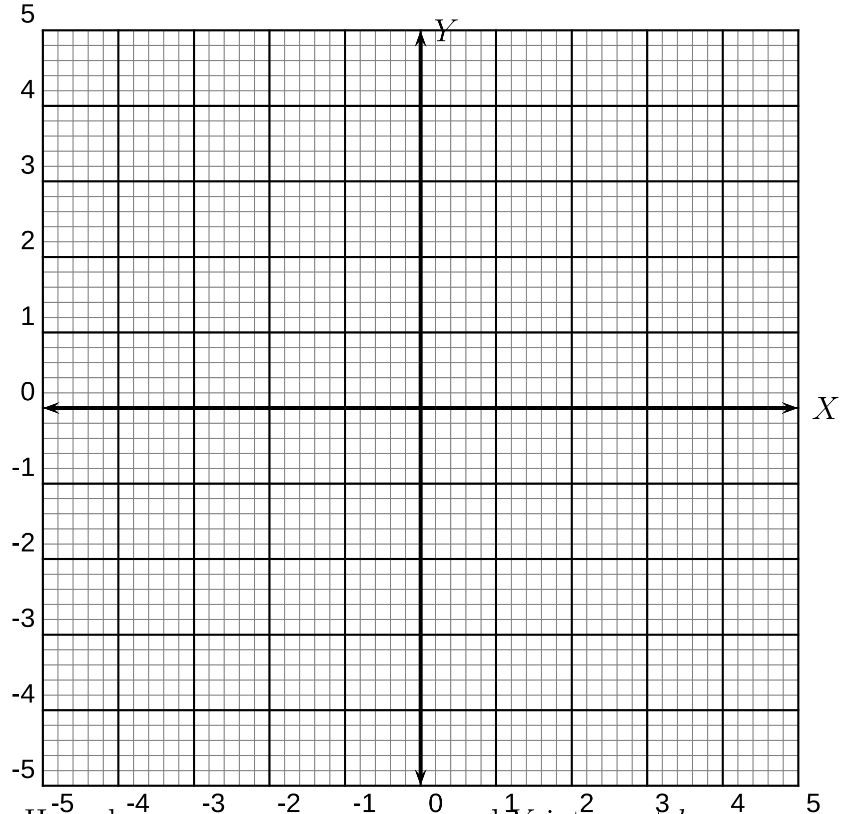
(b) When $x = 1$, $y =$ ________

(c) Slope of the line using points (0,   ) and (1,   ) is ________.

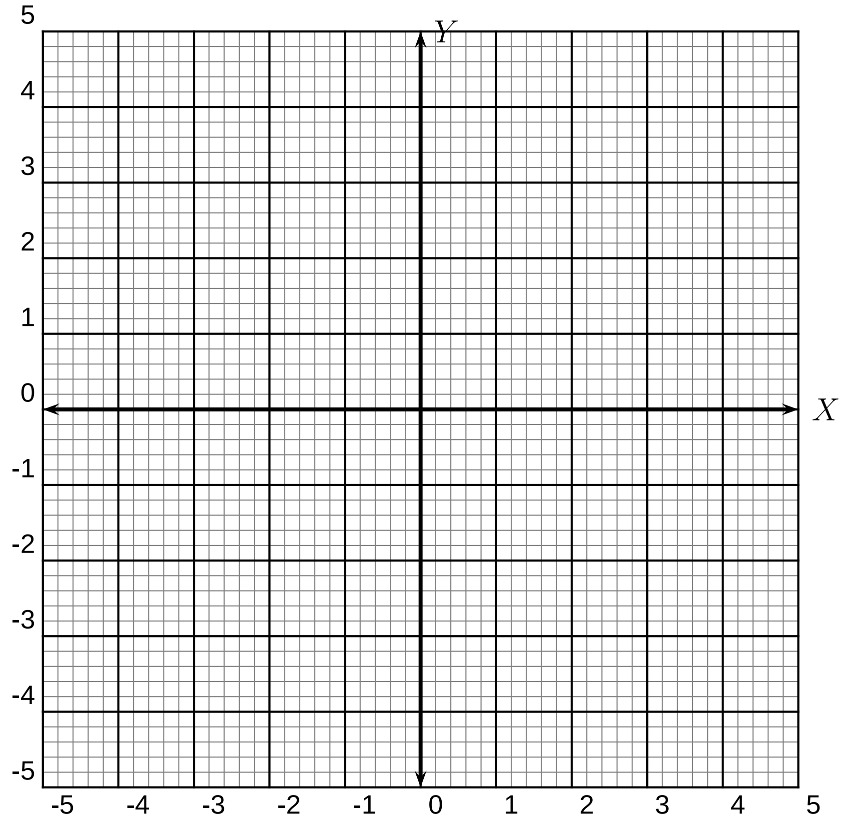
(d) The Y-intercept of the line is ________.

(6) Identify the slope and Y-intercept of the graphs of the following linear equations. Using their Y-intercepts and slopes, draw the lines.

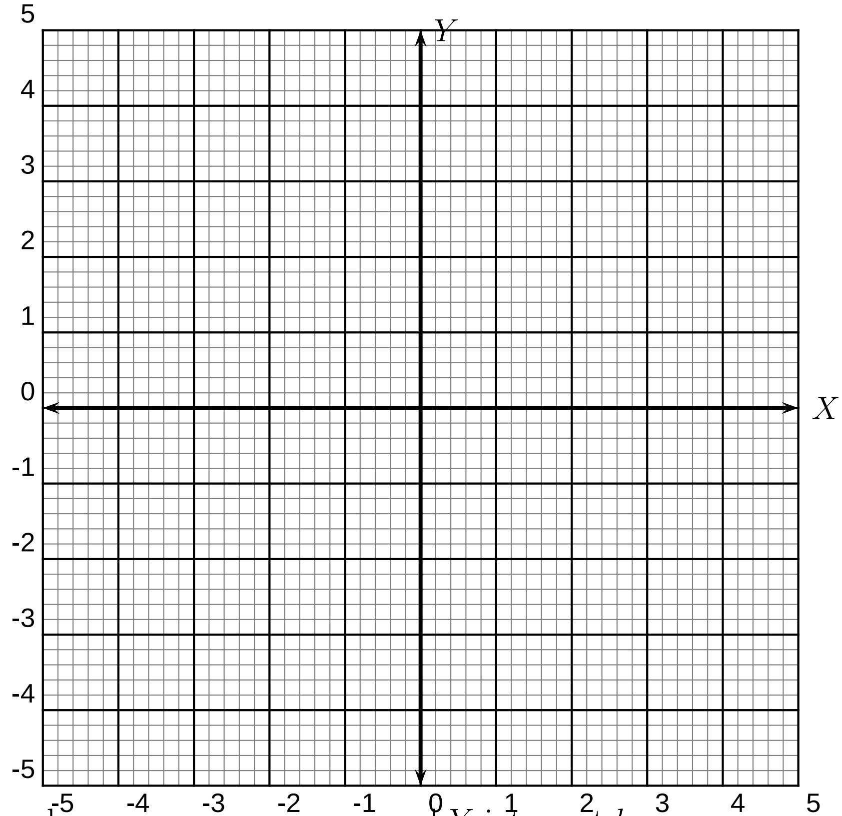
(a) $y = 3x + 1$. Here, slope $m =$ ____________ and Y-intercept $b =$ ____________.



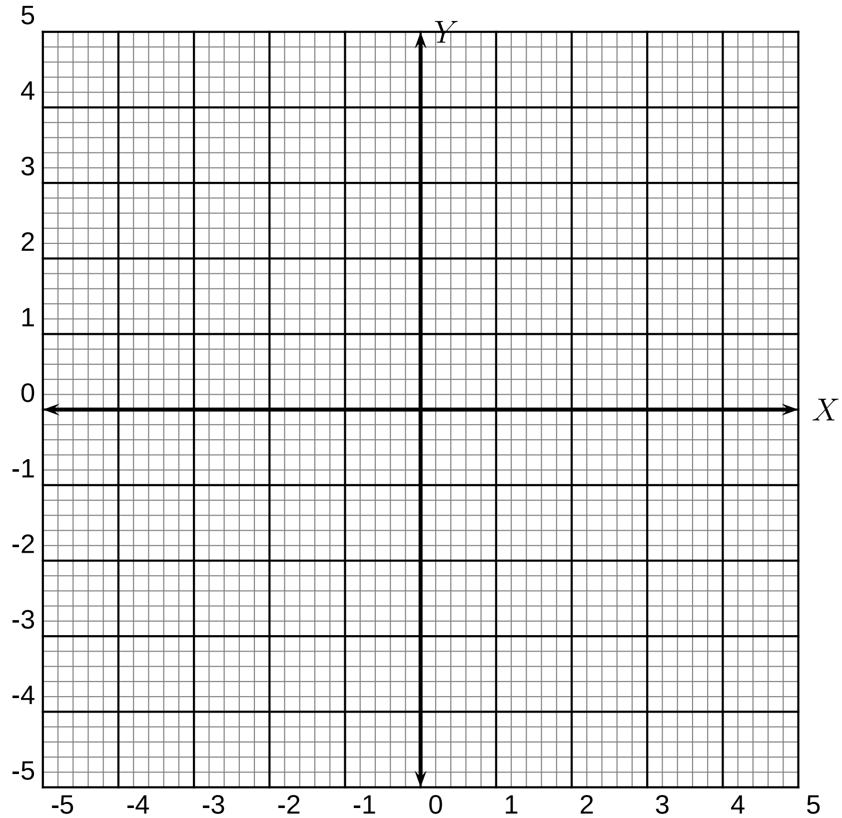
(b) $y = -5x + 2$. Here,slope $m =$ ____________ and Y-intercept $b =$ ____________.



(c) $y = 3x$. Here, slope $m =$ ____________ and Y-intercept $b =$ ____________.



(d) $y = -3$. Here, slope $m =$ ____________ and Y-intercept $b =$ ____________.



15. Equation of a line

(1) What is the slope-intercept form of a linear equation? (Why is its name **slope-intercept**?)

(2) What is the point-slope form of a linear equation? (Why is its name **point-slope**?)

(3) What is the standard form of a linear equation?

(4) A line goes through points $(1, 1)$ and $(3, 2)$.
   (a) What is its slope?
   (b) Write an equation for the line using the point-slope form. (Use the point $(1, 1)$).
   (c) Write the equation obtained above in the slope-intercept form. What is its Y-intercept?
   (d) Write the equation obtained above in the standard form.

(5) A line goes through points $(2, -1)$ and $(-3, 2)$.

(a) What is its slope?

(b) Write an equation for the line using the point-slope form. (Use the point $(-3, 2)$).

(c) Write the equation obtained above in the slope-intercept form. What is its Y-intercept?

(d) Write the equation obtained above in the standard form.

(6) A few definitions first:

- **Two non-vertical lines with slopes m_1 and m_2 are parallel if $m_1 = m_2$.**
- **Two non-vertical lines with slopes m_1 and m_2 are perpendicular if $m_1 m_2 = -1$.**
- **Two vertical lines are parallel.**
- **A vertical line is perpendicular to a horizontal line.**

Do the following pairs of equations represent parallel lines, or perpendicular lines, or neither parallel nor perpendicular?

(a) $2x + 3y = 12$ $\qquad$ $3x - 2y = -1$

(b) $2x + 3y = 12$ $\qquad$ $4x + 6y = -1$

(c) $x + y = 2$ $\qquad$ $x + y = -1$

(d) $x + y = 2$ $\qquad$ $x - y = -1$

(e) $2x + 3y = 12$ $\qquad$ $3x - y = -1$

(f) $12x + 3y = 12$ $\qquad$ $3x - 2y = -1$

(7) Write the standard form of the equation for the line that satisfies the given condition:

(a) The line passes through the point $(1, 2)$ and is parallel to the line given by the equation $2x + 3y = 5$.

(b) The line passes through the point $(-1, -3)$ and is parallel to the line given by the equation $x + 2y = 5$.

(c) The line passes through the point $(0, 0)$ and is perpendicular to the line given by the equation $-5x + y = 8$.

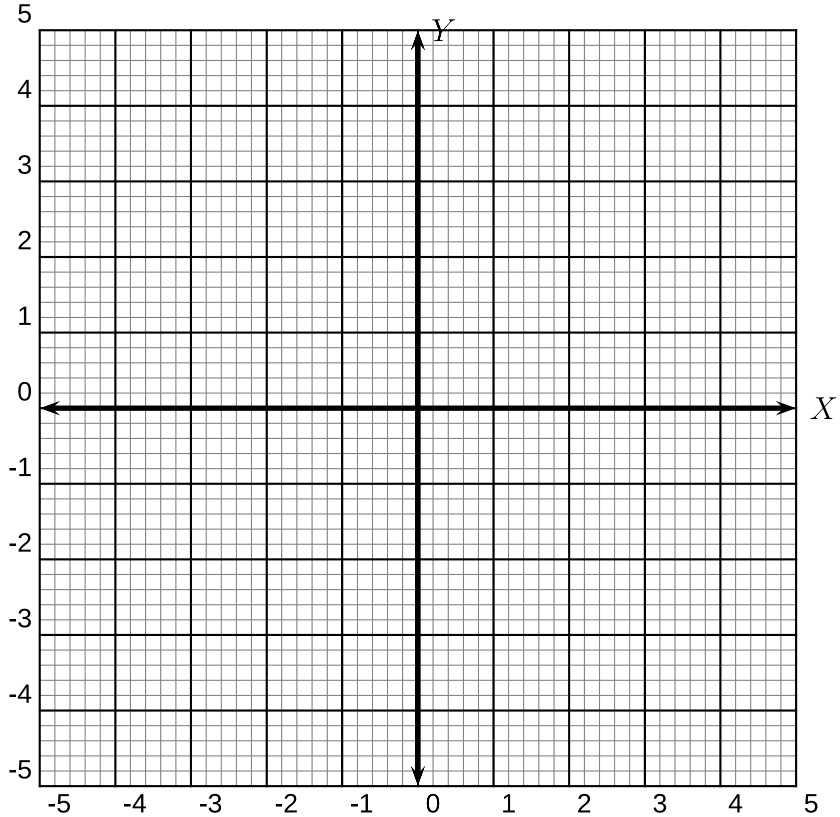
(d) The line passes through the point $(3, 7)$ and is perpendicular to the line given by the equation $x + 7y = 8$.

(e) The line passes through the points $(1, -3)$ and $(2, 2)$.

(f) The line has slope $\left(\frac{3}{2}\right)$ and passes through the point $(1, 3)$.

(g) The line has slope $\left(-\frac{4}{5}\right)$ and y-intercept $(0, -2)$.

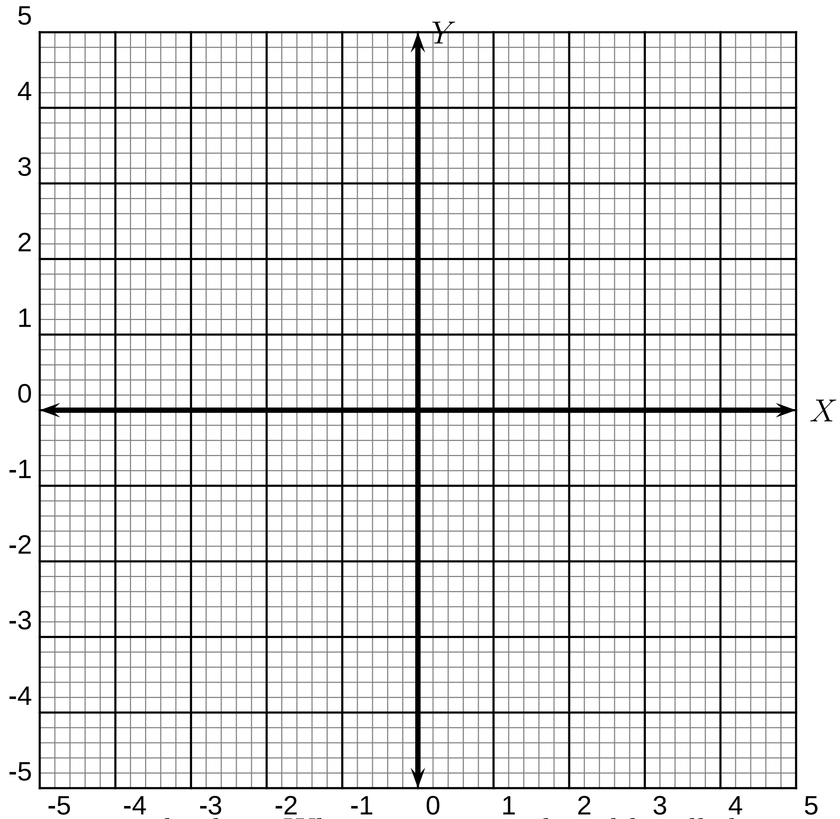
(8) On the given coordinate plane, draw a vertical line passing through the point $(3, 2)$.



(a) Pick any 5 points on this line. What property is shared by all these points?

(b) What is the equation of this line?

(9) On the given coordinate plane, draw a horizontal line passing through the point $(3, 2)$.



(a) Pick any 5 points on this line. What property is shared by all these points?

(b) What is the equation of this line?

(10) Summarize in your own words how you would draw graph using

- slope-intercept form of a linear equation

- standard form of a linear equation

- $x =$ a real number; say, $x = a$.

- $y =$ a real number; say, $y = b$.

(11) Summarize in your words which method you would use to write equation of a line if

- you are given the slope of the line and a point through which the line passes.

- you are given the slope of the line and the y-intercept.

- you are given a vertical line.

- you are given a horizontal line.

- you are given two points through which the line passes.

- you are given a point through which the line passes, and an equation of a line that is parallel to it.

- you are given a point through which the line passes, and an equation of a line that is perpendicular to it.

(12) A carpenter bills his customers \$40.00 an hour and a fixed carrying cost of \$ 100.
- Write an equation for the carpenter's charges if you need x hours of his time.

- How much would you have to pay if the carpenter is building a shelf and expects to spend 12 hours on it?

- How much time has the carpenter worked if he is getting paid \$3,310.00 for a particular job?

(13) Three years after an expansion, a company had sales of \$ 45,000. Eight years after the expansion, the sales were \$ 102,000. Assuming that the sales in dollars S and the time t in years are related by a linear equation, find the equation relating S and t.

(14) Find five integer solutions to the equation $3x - 2y = 2$.

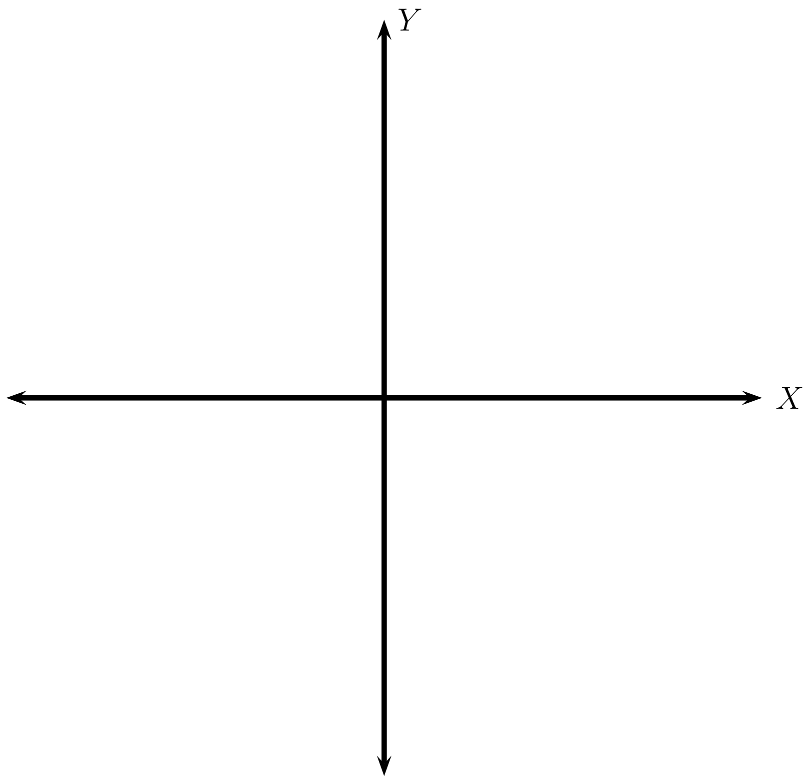
(15) Suppose you own a factory that prints notebooks. The cost (C) in dollars of printing x (in hundreds) of notebooks is given by the equation $C = 45x + 300$.

- What does the number 300 signify?
- What does the number 45 signify?
- Find the cost of printing 600 books.
- How many books may be printed if you are allowed to spend \$ 1200?

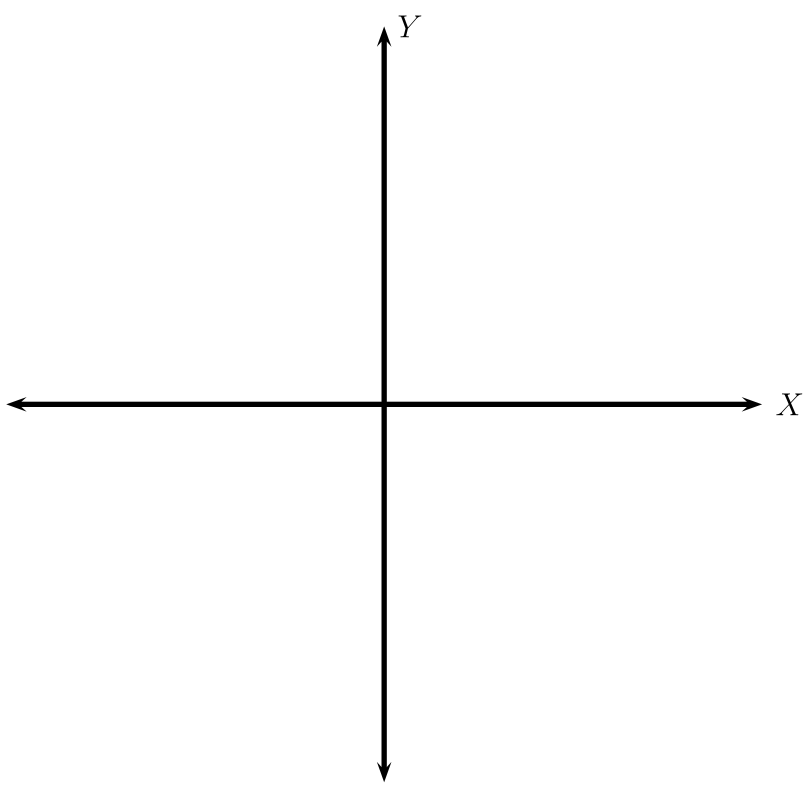
(16) A particular car is purchased for \$ 35,000. A year later, the car is worth only \$ 30,650. If the value of the car depreciates at the same rate, determine how long it will take the car to depreciate to \$ 8,900.

16. Graphing linear inequalities in two variables

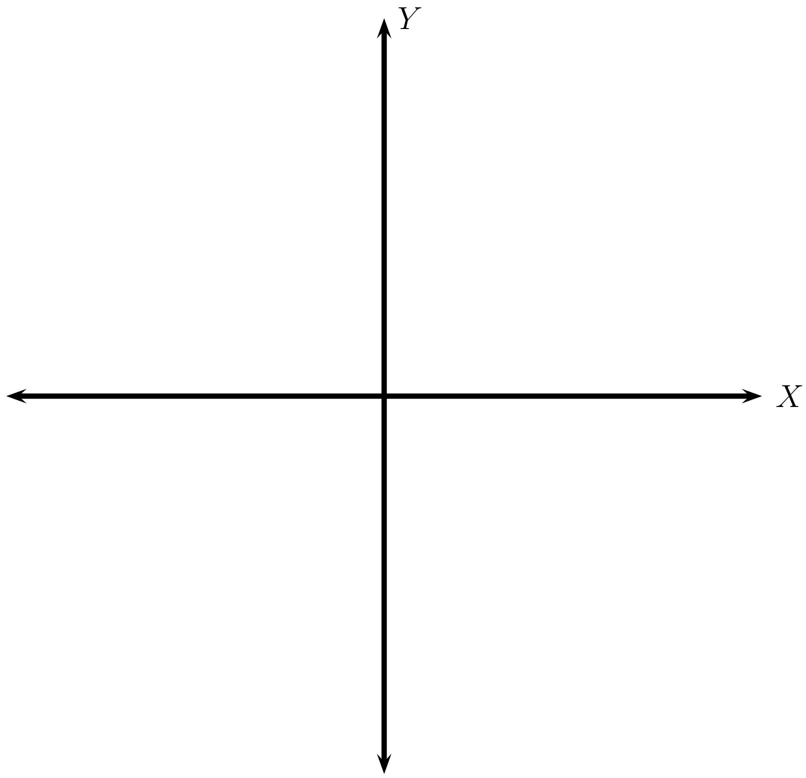
(1) Shade the region $-x + 3y \leq 9$.



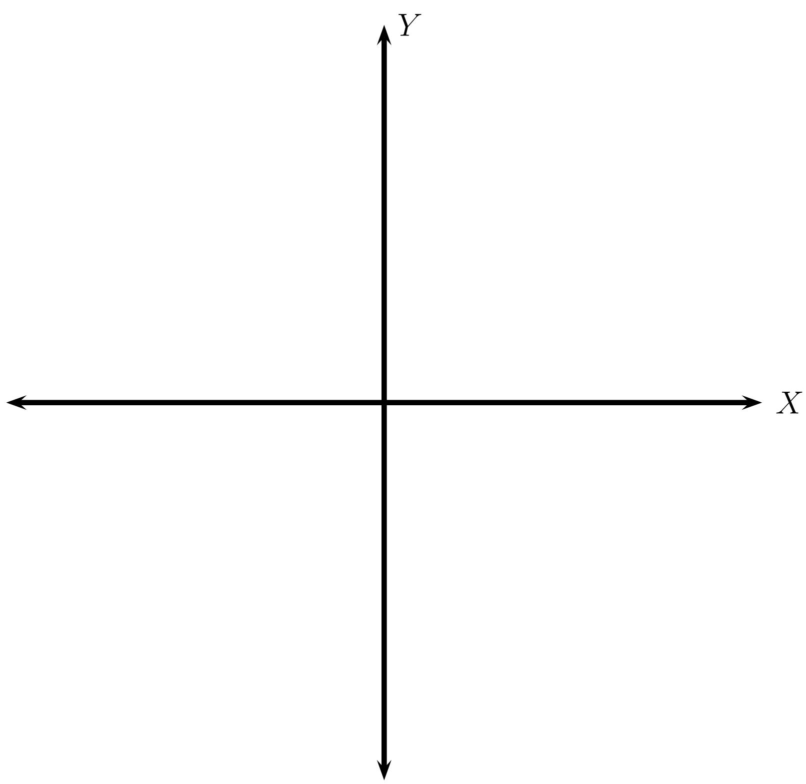
(2) Shade the region $3x + 4y \geq 9$



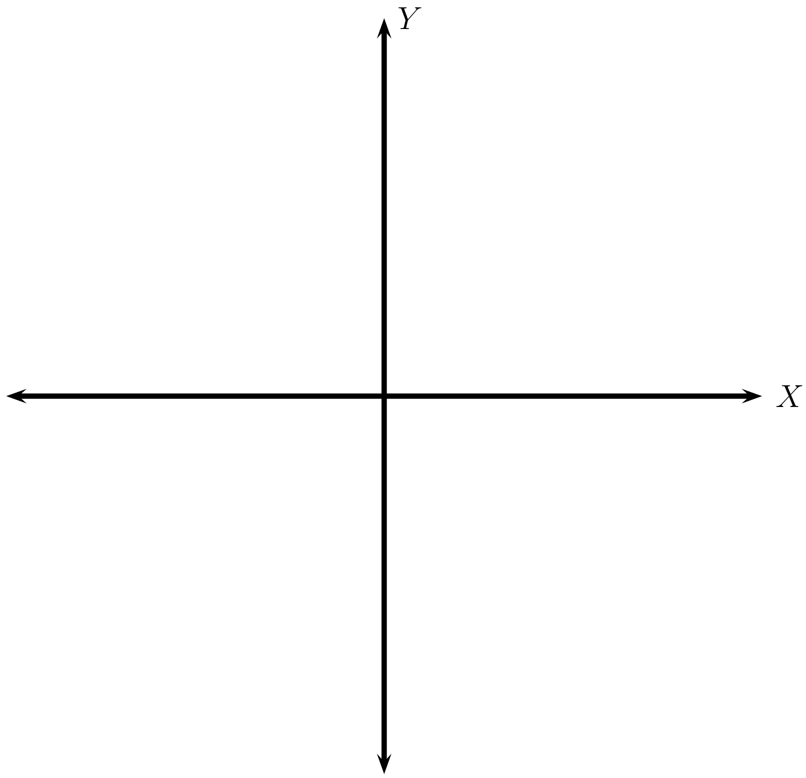
(3) Shade the region $4x - 7y < 12$.



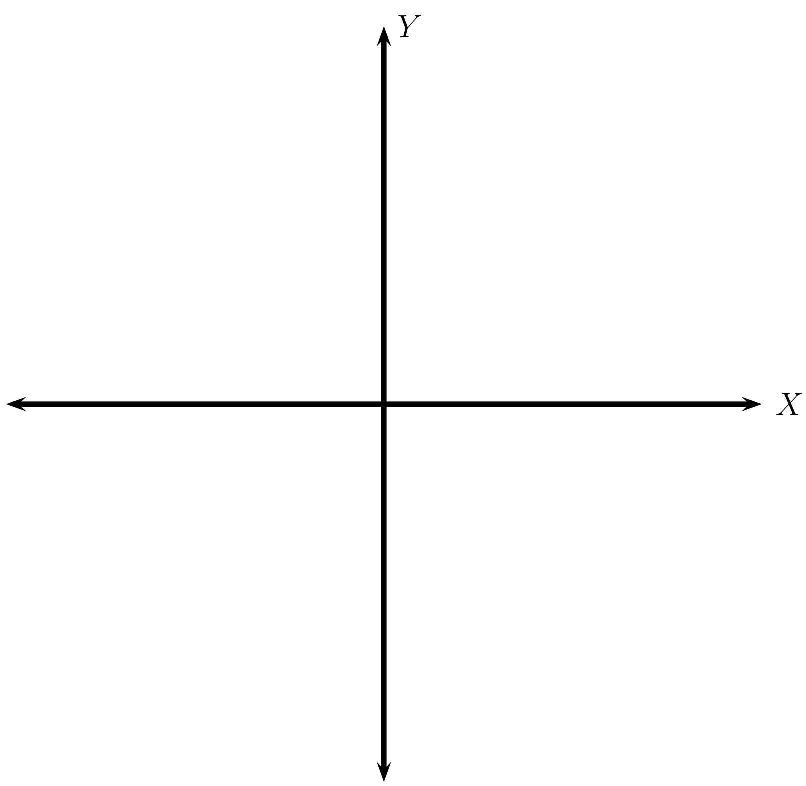
(4) Shade the region $2x + 5y < 18, \quad x > 0, \quad y > 0$.



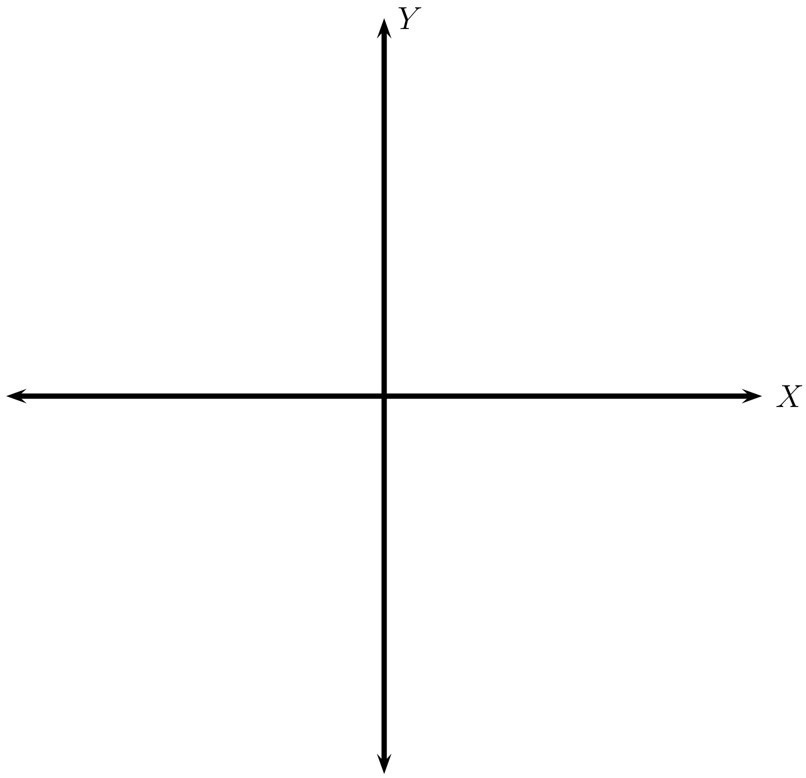
(5) Shade the region $3x + 2y \geq 5$.



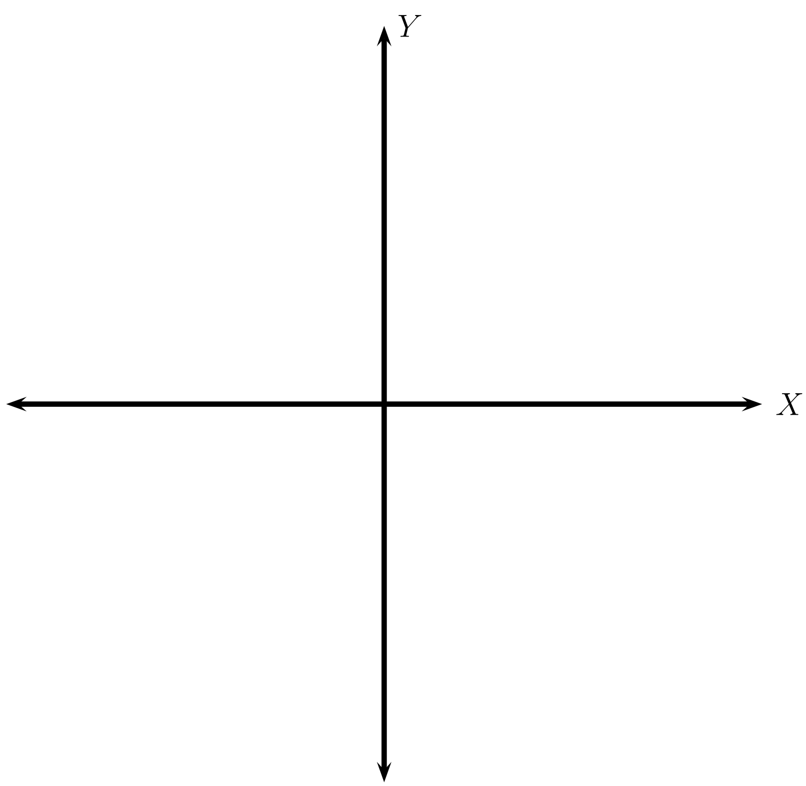
(6) Shade the region $x - y < 1, \quad x + y < 5, \quad x \geq 0$.



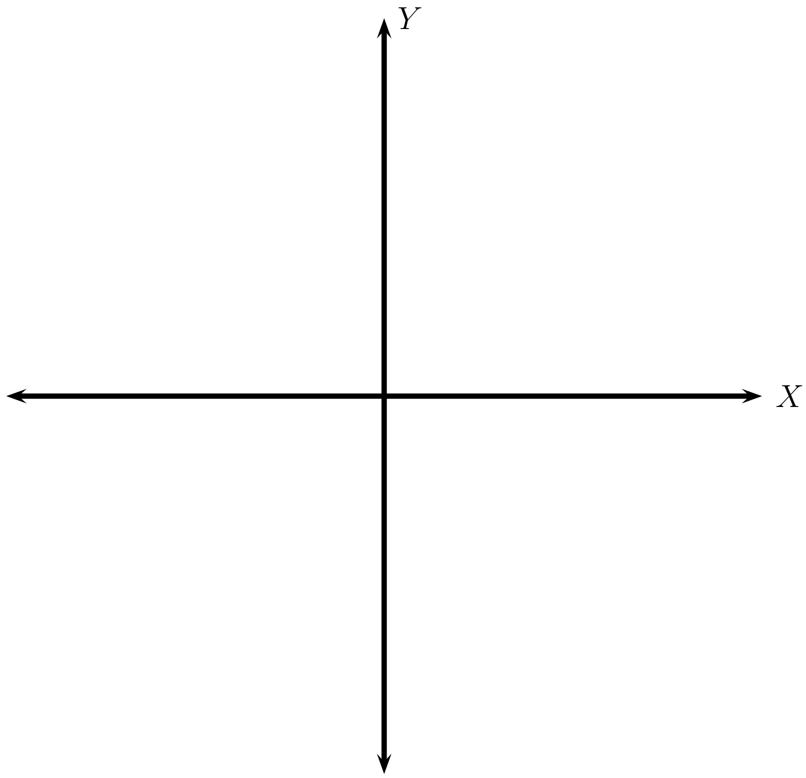
(7) Shade the region $x - y \geq -4, \quad x + y \leq 4, \quad y \geq 0.$



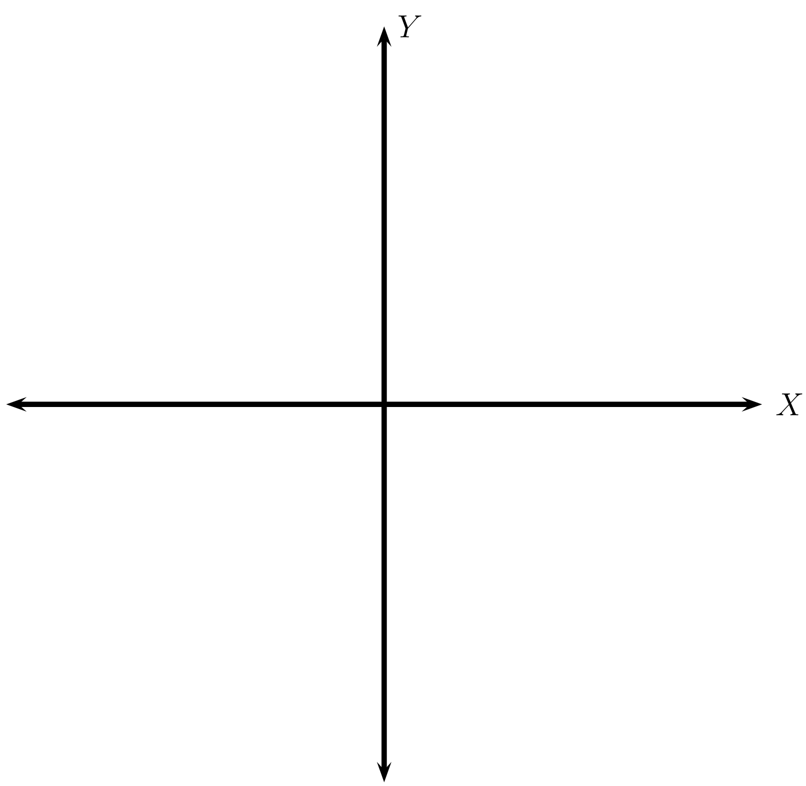
(8) Shade the region $x \geq 1, \quad y \geq 1.$



(9) Shade the region $1 \leq x < 4, \quad 2 < y \leq 6$.



(10) A gardner wants to apply fertilizers to his garden. Type A fertilizer has 1% Nitrogen and 2% Phosphorus. Type B fertilizer has 2% Nitrogen and 1% Phosphorus. The soil needs at least 4 pounds of Nitrogen and 5 pounds of Phosphorus. Graph the feasible region for these constraints.



17. Solving systems of linear equations in two variables by the addition method

(1) An example of a linear equation in 2 variables is $3x - 4y = 2$. An example of a **system** of two linear equations in 2 variables is

$$3x - 4y = 2$$
$$5x + y = 1$$

Give 5 examples of systems of two linear equations in 2 variables.

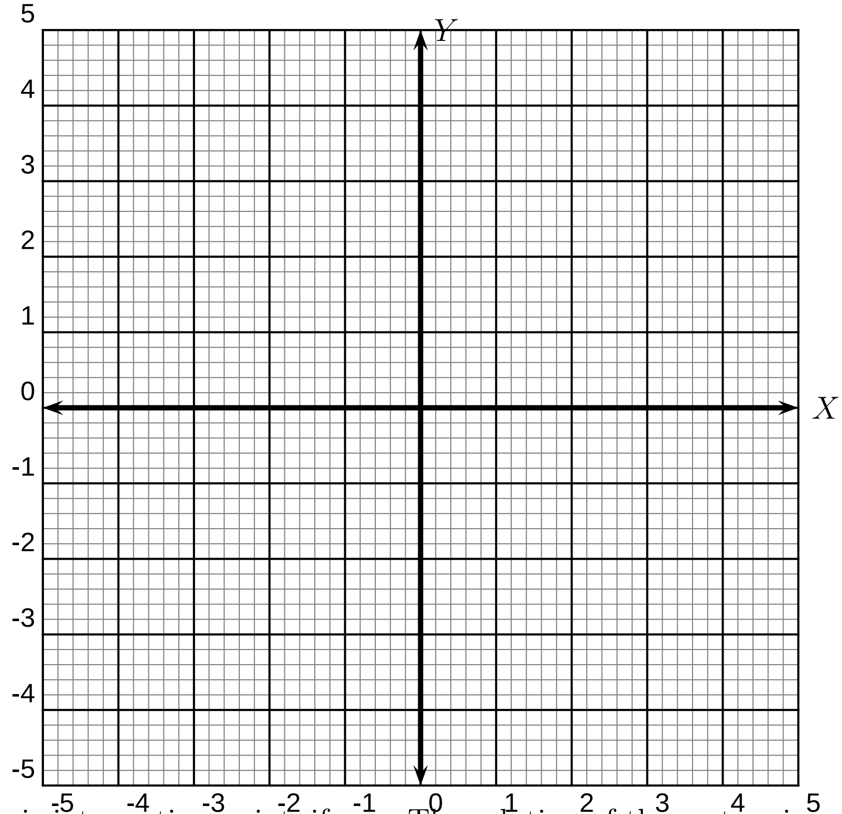
(2) Check whether any of the points $(0, 0), (1, 2), (1, -1), (1, 1)$ are solutions of the systems you gave in the previous question.

(3) We can solve systems of equations by graphing. For example, solve for x and y:

$$x + y = 3$$

$$-x + y = -1$$

- First, on the given coordinate plane, draw the graph of the two equations (remember, the graph of a linear equation is a line – so we just need two points).

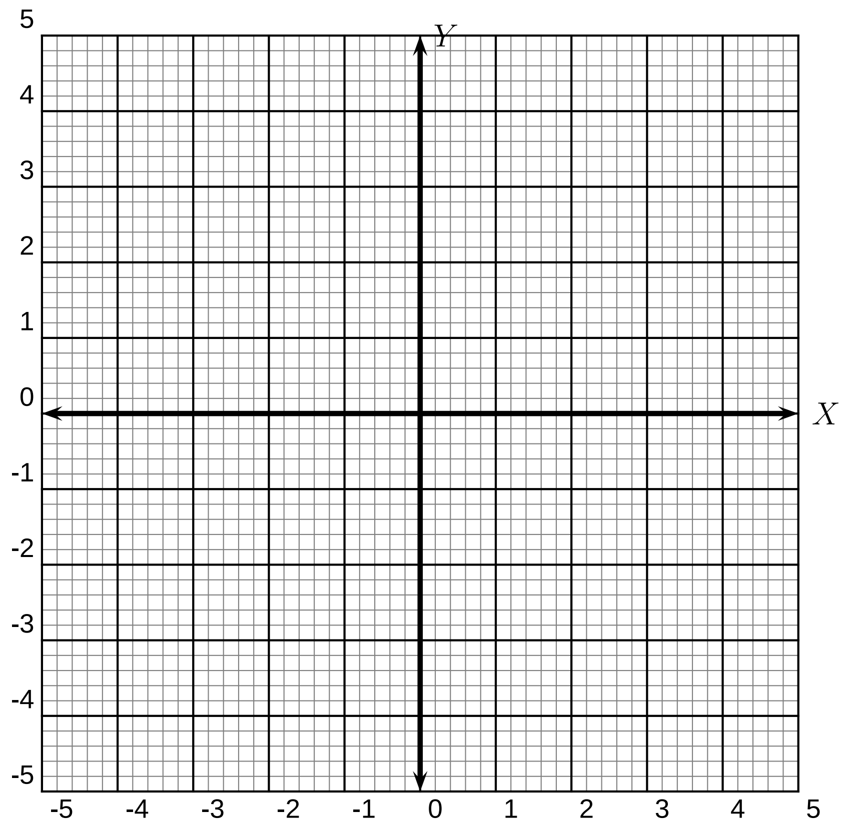


- Next, find their intersection point, if any. The solution of the system is $x =$ ____________, $y =$ ____________.
- Lastly, check whether you have indeed found the correct solution.

(4) Solve the following systems of equations by graphing:

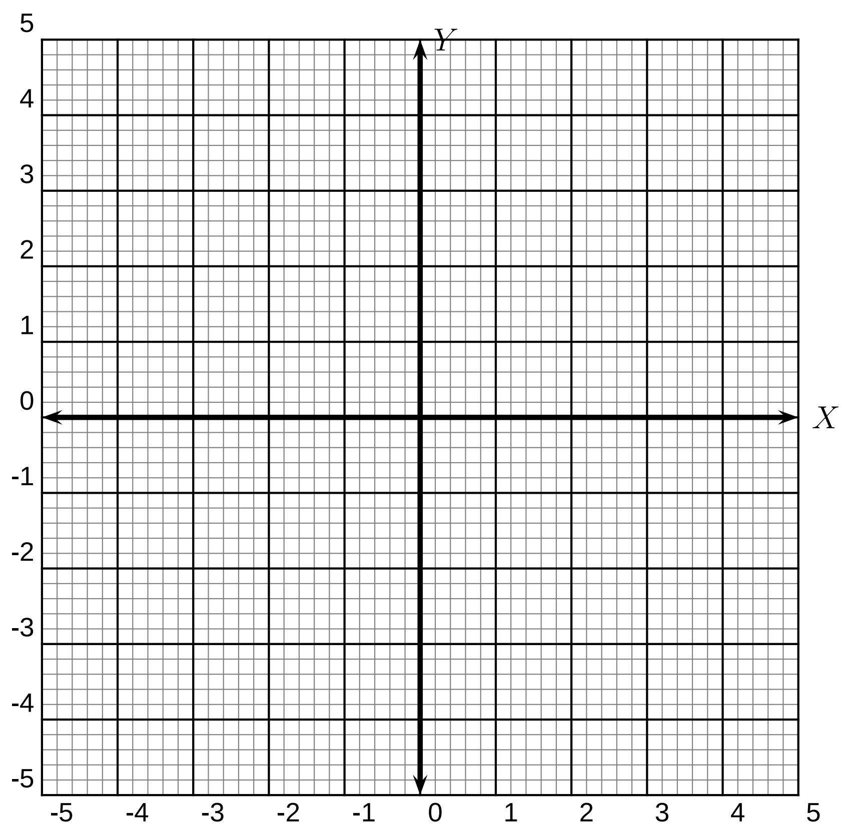
(a) System :

$$x - 2y = -4$$
$$3x + 6y = 0$$



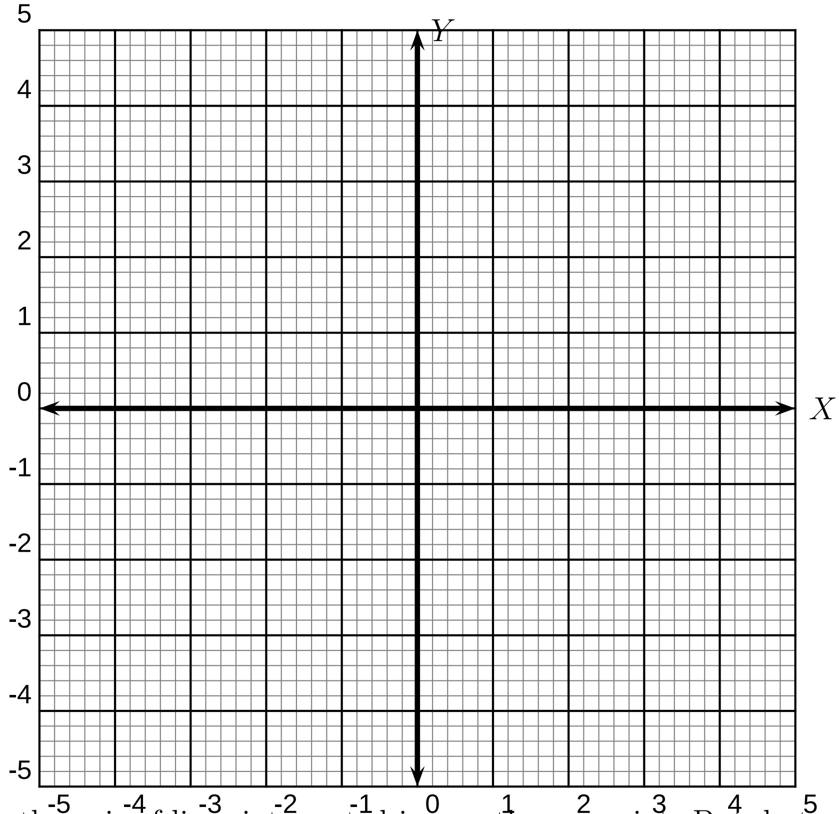
(b) System :

$$3x - 3y = -15$$
$$2x + y = -4$$



(c) System :

$$6x - 4y = -4$$
$$3x + 2y = -10$$

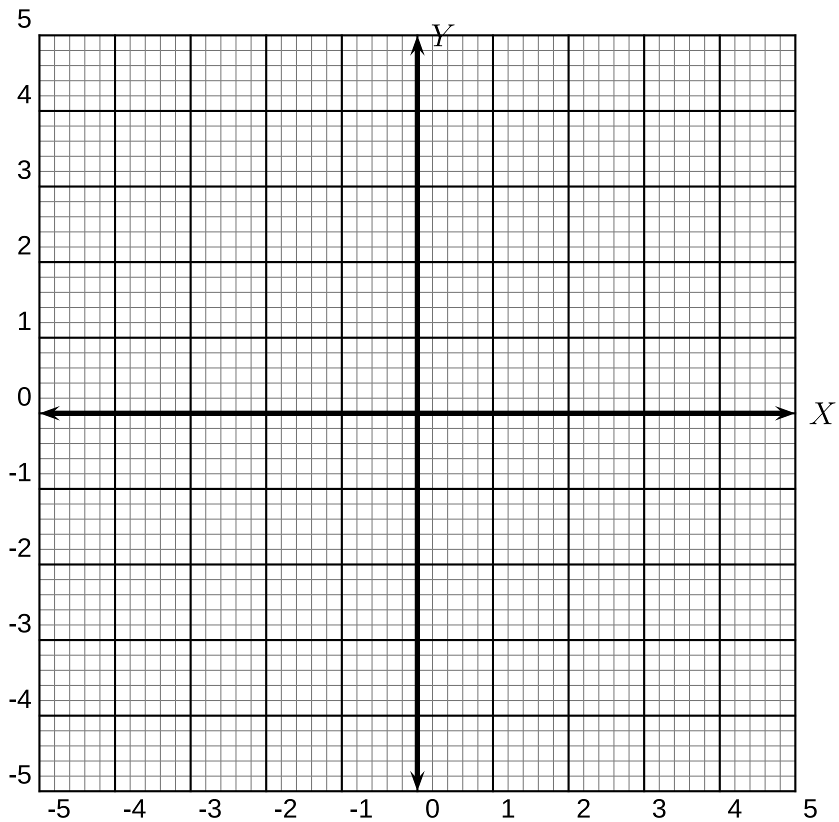


(5) In all of the above cases, the pair of lines intersected in exactly one point. But do two lines **always** intersect in exactly one point? Explain.

(6) Solve the following systems of equations by graphing:

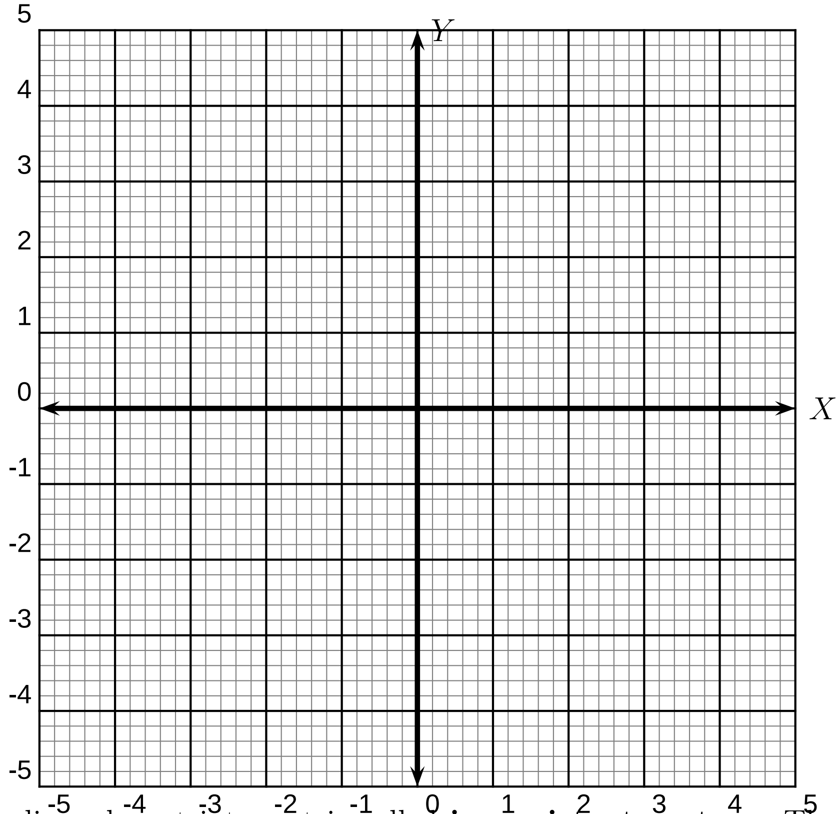
(a) System :

$$2x + 3y = 6$$
$$2x + 3y = 12$$



(b) System :

$$3x - 2y = 6$$
$$-3x + 2y = 12$$

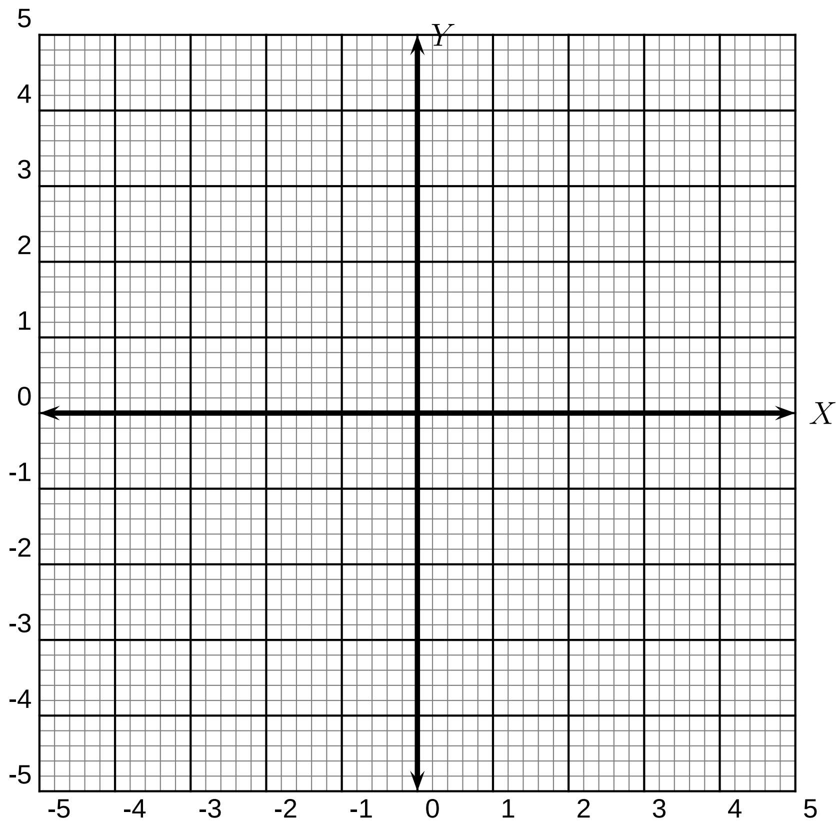


A system where the lines do not intersect is called **inconsistent system**. There are no solutions for inconsistent systems.

(7) Solve the following systems of equations by graphing:

(a) System :

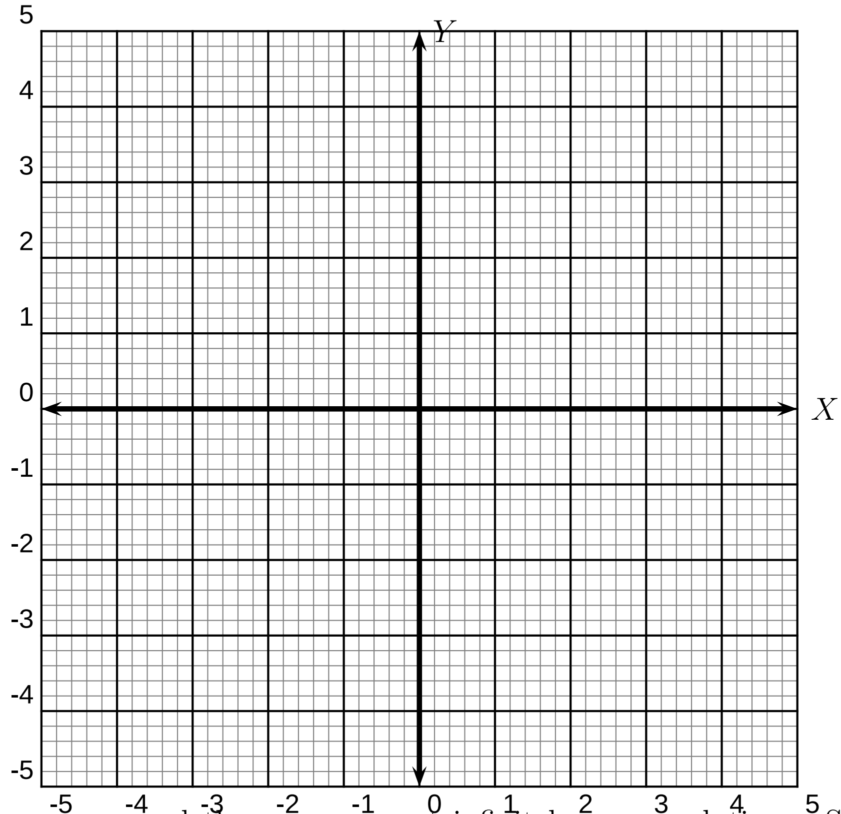
$$x + 3y = 3$$
$$2x + 6y = 6$$



(b) System :

$$-3x + 2y = 12$$

$$x - \left(\frac{2}{3}\right) y = -4$$



When the two lines are one and the same, we get infinitely many solutions. Such a system is called **dependent**. The solutions of a dependent system are to be described using one variable.

(8) Summarize in your own words, the three cases you encountered in solving a system of linear equations in 2 variables by graphing.

(9) The graphing method is not always a feasible method, as the intersection point may not have integral coordinates. So, we have some algebraic methods of solving systems of equations. One such method is the **addition** method (also called, the **simultaneous** method, or the **elimination** method). For example, solve for x and y:

$$2x - y = -2$$
$$3x - 2y = -5$$

- Multiply the first equation by 3 and the second equation by 2. Write down your results one below the other. (Why did we choose these numbers?)

- Now subtract the new second equation from the new first equation (Why?). Does it simplify?

- Substitute the y value in any of the original two equations to solve for x.

- Lastly, check whether you have indeed found the correct solution.

(10) Solve the following systems of equations by the addition method:

(a) $-3x + 2y = 1$
$4x + 3y = 2$

(b) $2x + 3y = -9$
$3x - 2y = -7$

(c) $4x + 3y = 6$
$2x + 5y = -4$

(11) In all of the above cases, we found exactly one ordered pair as the solution. Will we **always** find exactly one solution? Explain.

(12) Solve the following systems of equations by the method of addition:

(a) $2x + 3y = 6$
$2x + 3y = 4$

(b) $3x - 2y = 6$
$-3x + 2y = 8$

A system which gives a **false** statement is called an **inconsistent system**. There are no solutions for an inconsistent system.

(13) Solve the following systems of equations by the method of addition:

(a) $x + 3y = 2$
$2x + 6y = 4$

(b) $-3x + 2y = 12$
$x - \frac{2}{3}y = -4$

A system which gives a **true** statement when the variables are eliminated is called **dependent**. Such a system has infinitely many solutions and the solutions of a dependent system are to be carefully described using one variable.

(14) Summarize in your own words, the three cases you encountered in solving a system of linear equations in 2 variables by method of addition.

(15) State clearly and in **complete sentences** what your variables stand for.

(a) The sum of the measures of two angles is 141^o and their difference is 3^o. Find the two measures.

(b) The perimeter of a rectangle is 76 cm. If the difference between the length and the width is 4 cm., then find the length and the width.

(c) A fraction has value $\frac{5}{6}$. If 5 is added to the numerator and 2 is added to the denominator, then the new fraction has value 1. What is the original fraction?

(d) In a grocery store, 8 pounds of sugar and 4 pounds of flour cost me \$7.00 whereas, 1 pound of sugar and 2 pounds of flour cost me \$2.00. What is the price per pound of sugar and flour.

(e) Suppose you spend \$7.42 at a post-office to buy 13 stamps. If you bought 39-cent and 86-cent stamps, then how many of each kind did you buy?

18. Introduction to polynomials

Recall the **rules of exponents**.

$$
\begin{aligned}
x^a &= x \cdot x \cdot x \cdot \ldots \cdot x \qquad \textit{here } x \textit{ appears atimes} \\
x^a \cdot x^b &= \\
(x^a)^b &= \\
\textit{For } x \neq 0, \frac{x^a}{x^b} &= \\
\textit{For } x \neq 0, \quad x^0 &= \\
(xy)^a &=
\end{aligned}
$$

Simplify the following:

(1) x^0
(2) x^1
(3) 10^0
(4) $(N^3)^4$
(5) $(M^{100})^0$
(6) M^5M^6
(7) N^5M^6
(8) $(N^2M^3)^{12}$
(9) $\dfrac{N^{12}}{N^3}$
(10) $\dfrac{N^{12}M^7}{N^4M^6}$
(11) $\dfrac{N^{12}M^7 + N^5M^8}{N^4M^6}$ (**Hint :** Division distributes)
(12) $\dfrac{N^4M^6}{N^{12}M^7 + N^5M^8}$
(13) $\dfrac{N^{28}M^{37} - N^{52}M^{18}}{N^{14}M^{16}}$
(14) $N^3 \cdot N^5 \cdot N^{12} \cdot N^0$
(15) $\dfrac{x^4 \cdot x^8 \cdot x^0 \cdot x^{10}}{x^2 \cdot x^9 \cdot x^1}$

Recall the **distributive law** of multiplication.

$$a \cdot (b + c) = a \cdot b + a \cdot c$$
$$a \cdot (b - c) = a \cdot b - a \cdot c$$

(1) $2x(3x + 5y)$
(2) $2x(4x - 10y)$
(3) $-a^2(12a^3 + 8a - 7b)$

(4) $-5a^2b^3(-a^3b - a^2b^4 - ab)$

(5) $(a^{12}b^3 - 15ab)a^3b^2$

(6) $ab(a^2b^3 + a^3b^2)$

(7) $(N^2M^3 - NM^2 - 2N^0M^5)NM$

(8) $\xi(a^2b - ab^2 + a^2b^2)$ (Do not let ξ trouble you. It is no different from any other variable.)

(9) $(a + b)(a^2b - ab^2 + a^2b^2)$

(10) $(a + b + c)(a^2b - ab^2 + a^2b^2)$

(11) $(a^2 + ab + b^2)(a^2b - ab^2 + a^2b^2)$

(12) $(a^2 - ab - b^2)(a^2b + ab^2 - a^2b^2)$

Here we work with **polynomials**. A **polynomial in** x is an algebraic expression whose terms are of the form ax^n where a can be any real number and n can be any whole number.

(1) What is a monomial? Give 5 examples.

(2) What is a binomial? Give 5 examples.

(3) What is a trinomial? Give 5 examples.

(4) Give a polynomial in variables a, b, c, d, e each.

(5) What is a polynomial in variables x, y? Give 5 examples.

(6) What is the degree of a term? Explain using 5 different examples.

(7) What is the degree of a polynomial? Explain using 5 different examples.

(8) Write a polynomial with 5 unlike terms in variable x. Then write it in descending powers of x.

(9) Write a polynomial with 5 unlike terms in varibles m, n. Then write it in descending powers of m.

(10) If the expression is a polynomial, then write the degree of each of the terms, and find the degree of the polynomial.

(a) $4x^2 - 6x + 20 + 30x^3$

(b) $5x^2 - 3x + 1 - \dfrac{1}{x}$

(c) $5x^2 - 3x + 1 + \sqrt{x}$

(d) $5x^2 - 3x + 1 - abc$

(e) $20 - 32x^2 + 11xy^4 - 15x^2y^7 + 17x^3y^3z$

(f) 3

(g) $3x^{-1}$

(11) Give a polynomial in variable x of

(a) degree 3 with 4 unlike terms.

(b) degree 5 with 3 unlike terms.

(c) degree 30 with 2 unlike terms.

(d) degree 2.

(e) degree 1.

(f) degree 0.

(g) degree 100 of 1 term.

(12) Write each polynomial in descending powers of the indicated variable.

(a) $-3x + 10 - 5x^2 + 10x^5$. Variable x.

(b) $-11 + 13x + 14y - 2x^2 + xy - 4y^2 + 2x^3y^3 + x^6$. Variable y.

(c) $20 - 32x^2 + 11xy^4 - 15x^2y^7 + 17x^3y^3z$. Variable y.

(13) Add or subtract polynomials as indicated:

(a) $(3x + y) + (5x - 7y)$

(b) $(3x + y) - (5x - 7y)$

(c) $(3) + (2 + 5x)$

(d) $(3) - (2 + 5x)$

(e) $(20x^5y^3 + 12x^4y^3 - 13xy) + (2x^4y^3 - 12xy + 2x - 3y - 8)$

(f) $(20x^5y^3 + 12x^4y^3 - 13xy) - (2x^4y^3 - 12xy + 2x - 3y - 8)$

Multiply the polynomials:

(1) $(3x + y)(5x - 7y)$

(2) $(3)(2 + 5x)$

(3) $(a + b)(a^2 - ab + b^2)$

(4) $(a - b)(a^2 + ab + b^2)$

(5) $(a + b)(a - b)$

(6) $(a + b)^2$

(7) $(a - b)^2$

(8) $(x + y)(x^2 + 2xy + y^2)$

(9) $(x-y)(x^2-2xy+y^2)$

(10) $(x+y)(x^2-2xy+y^2)$

(11) $(x-y)(x^3+3x^2y+3xy^2+y^3)$

(12) $(x+y)^3$

(13) $(x-y)^3$

(14) $(20x^5y^3+12x^4y^3)(2x^4y^3-12xy+2x-3y-8)$

Verify the following formulae:

(1) $(a+b)(a-b)=a^2-b^2$
(2) $(a+b)^2=a^2+2ab+b^2$
(3) $(a-b)^2=a^2-2ab+b^2$
(4) $(a+b)^3=a^3+3a^2b+3ab^2+b^3$
(5) $(a-b)^3=a^3-3a^2b+3ab^2-b^3$
(6) $(a^3+b^3)=(a+b)(a^2-ab+b^2)$
(7) $(a^3-b^3)=(a-b)(a^2+ab+b^2)$

Perform the following simple divisions:

(1) $(3x^5 - 14x^4 + 3x^3 - 2x) \div (2x)$

(2) $(-12x^3y^2 + 8x^3y + 9x^2y^3 + 11) \div (2)$

(3) $(-12x^3y^2 + 8x^3y + 9x^2y^3 + 11xy) \div (2xy)$

(4) $(2y^2 + 3y - 11) \div (2x)$

(5) $\dfrac{3x^2m^2 - 4xm^2 + 4xm - 10}{mn}$

19. Factoring polynomials

Factor each expression completely.

(1) $2x - 20$

(2) $22x^2 - 18x^3y$

(3) $12x(a+b) - 28y(a+b)$

(4) $3x(x+5) - 7(x+5)$

(5) $4x(2x-3y) + y(2x-3y)$

Factor by the grouping method:

(1) $14ac + 6ad + 35bc + 15bd$

(2) $4rt + ru - 12st - 3su$

(3) $54ac + 66ad - 9bc - 11bd$

(4) $6pr - 15ps - 8qr + 20qs$

Difference of Squares:

(1) $a^2 - b^2$

(2) $x^2 - y^2$

(3) $\alpha^2 - \beta^2$

(4) $(2x)^2 - (3y)^2$

(5) $4a^2 - 9b^2$

(6) $16a^3 - 25ab^2$

(7) $12x^2y^4 - 27x^4y^2$

(8) $3a^2 - 5b^2$

(9) $x^8 - y^4$

(10) $x^4y^8 - a^4b^4$

Factorize the following:

(1) $x^2 + 5x + 6$

(2) $x^2 - 5x + 6$

(3) $x^2 + 7x + 6$

(4) $x^2 - x - 6$

(5) $x^2 + 5x - 6$

(6) $x^2 + x - 6$

(7) $x^2 - 5x - 6$

(8) $x^2 - 7x + 6$

(9) $a^2 + 13a + 30$

(10) $a^2 + 7a - 30$

(11) $a^2 - 7a - 30$

(12) $a^2 - 13a + 30$

(13) $a^2 - 13a - 30$

(14) $a^2 + 13a - 30$

(15) $a^2 + 19a + 18$

(16) $a^2 + 11a + 18$

(17) $a^2 - 19a + 18$

(18) $a^2 + 7a - 18$

(19) $a^2 + 17a - 18$

(20) $a^2 - 7a - 18$

(21) $a^2 - 17a - 18$

(22) $a^2 + 2ab - 24b^2$

(23) $x^2 - 12xy - 64y^2$

(24) $\alpha^4 + \alpha^2\beta^2 - 2\beta^4$

(25) $(x+y)^2 + 17(x+y)(a-b) + 30(a-b)^2$

(26) $(2x-y)^2 + 19(2x-y)(a+b) + 48(a+b)^2$

(27) $x^4 - 13x^2y^2 + 36y^4$

(28) $6a^2 + 23a + 20$

(29) $6a^2 + 34a + 20$

(30) $6a^2 + 7a - 20$

(31) $6a^2 - 23a + 20$

(32) $6a^2 + 2a - 20$

(33) $6a^2 - 34ab + 20b^2$

(34) $6a^2 - 7ab - 20b^2$

(35) $6a^2 - 2ab - 20b^2$

(36) $35h^2 - 87hk + 22k^2$

(37) $35h^2 + 117hk + 22k^2$

(38) $35h^2 + 67hk - 22k^2$

(39) $35h^2 + 103hk - 22k^2$

(40) $35h^2 - 67hk - 22k^2$

(41) $35h^2 - 103hk - 22k^2$

(42) $35h^2 - 117hk + 22k^2$

(43) $35h^2 + 87hk + 22k^2$

(44) $48a^4 - 82a^2b^2 + 35b^4$

(45) $48a^4 - 2a^2b^2 - 35b^4$

(46) $48a^4 + 2a^2b^2 - 35b^4$

(47) $48a^4 + 82a^2b^2 + 35b^4$

(48) $49h^4 + 70h^2k^2 + 25k^4$

(49) $49h^4 - 70h^2k^2 + 25k^4$

(50) $6(x+y)^2 + 31(x+y)(a-b) + 35(a-b)^2$

(51) $6(x-y)^2 - 31(x-y)(a+b) + 35(a+b)^2$

(52) $6x^4 - 35x^2y^2 + 25y^4$

Factorize the following completely:

(1) $16x^2 - 36y^2$

(2) $-15a^2 + 25b^2$

(3) $294a^3b - 54ab^3$

(4) $a^8 - b^8$

(5) $a^4 + b^4$

(6) $2h^3 - 8hk^2 + 3h^2k - 12k^3$

(7) $10x^3 + 15xy^2 + 6x^2y + 9y^3$

(8) $21a^3 - 14a^2b - 27ab^2 + 18b^3$

(9) $21a^3 - 14a^2b + 27ab^2 - 18b^3$

(10) $10x^3 - 15xy^2 - 6x^2y + 9y^3$

(11) $35xyh^2 + 117xyhk + 22xyk^2$

(12) $35h^3 - 67h^2k - 22hk^2$

(13) $6a^3 + 17a^2b - 28ab^2$

(14) $6a^2b - 17ab^2 - 28b^3$

(15) $40x^3y - 7x^2y^2 - 3xy^3$

State whether the following is true or false. If false, then give the correct formula:

(1) $(a+b)^2 = a^2 + b^2$

(2) $(a-b)^2 = a^2 - b^2$

20. Solving equations by factoring

(1) What is a **linear equation**? Give 5 examples.

(2) What is a **quadratic equation**? Give 5 examples.

(3) Is the following argument correct? Explain your answer.

$$a = b$$
$$\textit{So } a^2 = ab$$
$$\textit{Thus, } a^2 - b^2 = ab - b^2$$
$$\textit{Hence } (a-b)(a+b) = b(a-b)$$
$$\textit{Dividing both sides by } (a-b) \textit{ we get } (a+b) = b$$
$$\textit{As } a = b, \textit{ we get } b + b = b$$
$$\textit{Thus, } 2b = b$$
$$\textit{Dividing both sides by } b \textit{ we get } 2 = 1.$$

(4) Is the following argument correct? Explain your answer.

$$x^2 - 5x + 6 = 12$$
$$(x-3)(x-2) = 12$$
$$(x-3) = 12 \textit{ or } (x-2) = 12$$
$$x = 12 + 3 \textit{ or } x = 12 + 2$$
$$x = 15 \textit{ or } x = 14.$$

(5) State the **Zero factor property**.

(6) State true or false (if false, then explain with examples):

If $a \cdot b = 24$ then $a = 24$ or $b = 24$.

(7) Solve each equation if possible.

(a) $x(x-2)(x+3)(x-5) = 0$

(b) $x(x-8)(x+11)(x-100)(x+2000) = 0$

(c) $x^2 - 2x = 0$

(d) $x^2 - 2x = 8$

(e) $a^3 - 4a^2 = 0$

(f) $x^3 - 4x^2 = -4x$

(g) $h^4 = 16$

(h) $h^5 = 16h$

(i) $x^5 = 4x$

(j) $15x^2 + 2x = 8$

(k) $14x^3 + 54x^2 = 8x$

(l) $-9x^2 = 30x + 25$

(m) $x - 4 = \dfrac{(2x-1)}{(x+4)}$

(n) $\dfrac{x}{2} = \dfrac{(x+12)}{(x-3)}$

(o) $x^2 - 2 = \dfrac{(x-6)}{6}$

(p) $4x^2 + 4x = \dfrac{15}{2} + 5x.$

(q) $(x-2)(x+2) + x = \dfrac{19x}{3}.$

(r) $-4 = \dfrac{2x(x-5)}{3}.$

(s) $-2x^2 = \dfrac{x-35}{3}.$

(t) $6x(x+1) = 20 - x.$

(u) $\dfrac{-7}{2}x^2 - \dfrac{7}{2} = 6x - 1.$

(v) $x^2 - 25 = 0$

(w) $x^2 - 16 = 0$

(x) $3x^2 - 12 = 0$

(y) $x^2 - 15 = 0$

(z) $2x^2 - 5 = 0$

(aa): $9x^2 - 1 = 0$

(ab): $7x^2 - 8 = 0$

(1) Can you solve for x in the following equations?

(a) $x^2 + 25 = 0$

(b) $x^2 + 15 = 0$

(c) $2x^2 + 5 = 0$

(d) $9x^2 + 1 = 0$

(2) Solve for x:

(a) $3x^2 + 5x = 0$

(b) $x^2 - 16x = 0$

(c) $x^2 - 25x = 0$

(d) $7x^2 + 5x = 0$

(e) $10x^2 + 11x = 0$

(1) Find the two numbers whose difference is 2 and product is 8.
Let the smaller number be x.
Then the larger number is ____________.
Their product is 8. So,

There are two solutions to this problem:
(a) The smaller number is ____________ and the larger number is ____________.
(b) The smaller number is ____________ and the larger number is ____________.

(2) The difference between two numbers is 3 and their product is 28. Find the two numbers.

(3) The difference between two numbers is 4 and their product is 12. Find the two numbers.

(4) The product of two consecutive even integers is 80. Find the two numbers.
Let the smaller even integer be x.
Then the larger even integer is ____________.
Their product is 80. So,

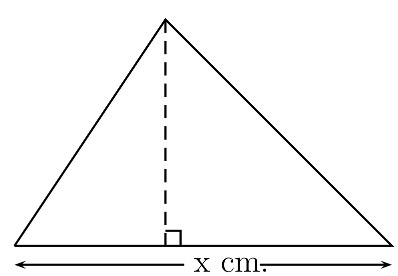
There are two solutions to this problem. The two consecutive even integers are ____________, ____________ or ____________, ____________.

(5) The product of two consecutive odd integers is 143. Find the two odd integers.

(6) The sum of the squares of two consecutive even integers is 340. Find the two even numbers.

(7) The sum of the squares of two consecutive odd integers is 394. Find the two odd numbers.

(8) The height of a triangle is 2 cm more than the length of its base. The area of the triangle is 12 cm^2. Find the length of the base.
Let the length of the base be x cm.
Then the height of the triangle is ________________ cm.
The area is 12 cm^2. So,



The length of the base is ________________ cm.

(9) The length of a rectangle is twice its width. If the area of the rectangle is 32 sq. in., find the length of the rectangle.

(10) One leg of a right triangle is 3 cm shorter than the other leg. If the area of the triangle is 14 sq. cm., then what are the lengths of the two legs? What is the length of the hypotenuse of the triangle?

(11) If the area of a circle is 49π sq. cm., then what is the circumference of the circle?

21. Roots and Radicals

(1) What is the principal square root?

(2) What is the radicand?

(3) Identify the radicand in each expression:

(a) $\sqrt{18}$

(b) $\sqrt{x^2+1}$

(c) $\sqrt{\dfrac{x}{y}}$

(4) What is a rational number? Give 5 examples.

(5) Find the square roots:

(a) $\sqrt{49}$

(b) $\sqrt{81}$

(c) $\sqrt{169}$

(d) $-\sqrt{\dfrac{49}{81}}$

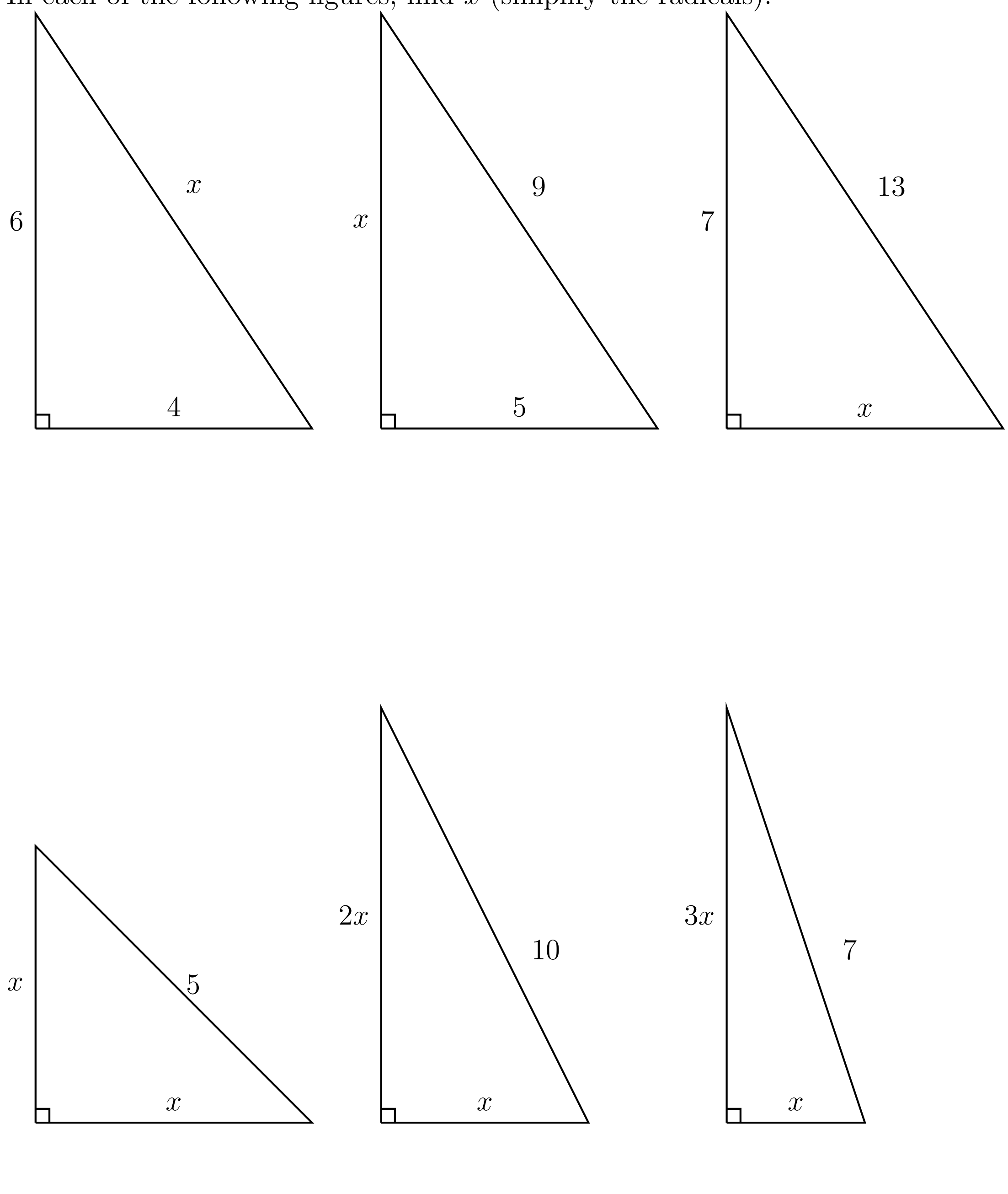
(6) Explain using 5 examples the statement, "If $x^n = a$ then $x = \sqrt[n]{a}$."

(7) Explain using 5 examples the statement, "$\sqrt[n]{x^n} = |x|$ if n is even."

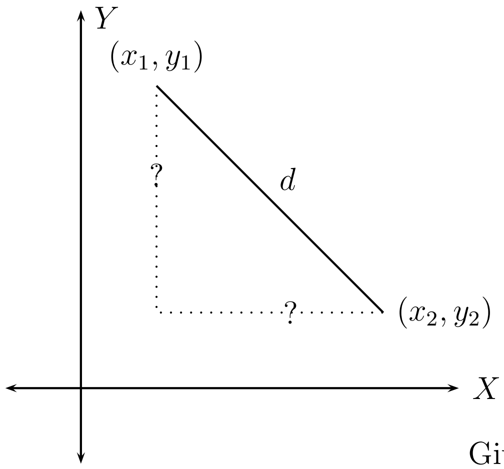
(8) Explain using 5 examples the statement, "$\sqrt[n]{x^n} = x$ if n is odd."

(9) Using an illustration state the Pythagorean theorem.

(10) In each of the following figures, find x (simplify the radicals):



(11) The distance formula:



Given two points (x_1, y_1) and (x_2, y_2) on the coordinate plane, the distance between them is given by

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

Find the distance between the following pairs of points:

(a) $(1, 2), (3, 4)$

(b) $(2, 2), (3, 3)$

(c) $(1, 1), (1, 1)$

(d) $(-1, 3), (2, -4)$

(12) Evaluate if possible

(a) $\sqrt[4]{2^4}$

(b) $\sqrt[4]{(-2)^4}$

(c) $\sqrt[3]{2^3}$

(d) $\sqrt[3]{(-2)^3}$

(e) $\sqrt[3]{-27}$

(f) $\sqrt[5]{32}$

(g) $\sqrt[5]{-32}$

(h) $\sqrt[7]{-128}$

(i) $\sqrt[4]{81}$

(j) $\sqrt{81}$

(k) $\sqrt{x^2}$

(l) $\sqrt[4]{x^4}$

(m) $\sqrt[5]{-243n^5}$

(n) $\sqrt[5]{\dfrac{-1024m^{10}n^{15}}{u^{20}}}$

(o) $\sqrt[3]{\dfrac{27m^{10}n^{15}}{u^{20}}}$

(p) $\sqrt{\dfrac{27x^5y^{13}}{z^{20}u^4}}$

(q) $\sqrt[4]{\dfrac{32a^{12}b^{11}}{c^9d^{23}}}$

(r) $\sqrt{x^{45}}$

(s) $\sqrt[3]{x^{45}}$

(t) $\sqrt[4]{x^{45}}$

(13) Complete the following table of square roots:

$\sqrt{0} =$	$= 6$	$= 12$	$= 18$
$\sqrt{1} =$	$= 7$	$= 13$	$= 19$
$\sqrt{4} =$	$= 8$	$= 14$	$= 20$
$\sqrt{9} =$	$= 9$	$= 15$	$= 30$
$= 4$	$= 10$	$= 16$	$= 40$
$= 5$	$= 11$	$= 17$	$= 50$

(14) Complete the following table of cube roots:

$\sqrt[3]{0} =$	$= 3$	$= 6$	$= 9$
$\sqrt[3]{1} =$	$= 4$	$= 7$	$= 10$
$\sqrt[3]{8} =$	$= 5$	$= 8$	$= 100$

Write down in your notebook the following **important** properties of square roots:

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}, \quad a \geq 0, b \geq 0$$
$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, \quad a \geq 0, b > 0$$
$$(\sqrt{a})^2 = a, , \quad a \geq 0.$$

Answer the following:

(1) Is $\sqrt{a^2 + b^2} = a + b$? Explain.

(2) Is $\sqrt{a^2 - b^2} = a - b$? Explain.

(3) Is $\sqrt{\dfrac{1}{a^2 + b^2}} = \dfrac{1}{a + b}$? Explain.

(4) Is $\sqrt{\dfrac{1}{a^2 + b^2}} = \dfrac{1}{a} + \dfrac{1}{b}$? Explain.

(5) Simplify each square root:
- (a) $\sqrt{45}$
- (b) $\sqrt{72}$
- (c) $\sqrt{192}$
- (d) $\sqrt{121 + 100}$
- (e) $\sqrt{121} + \sqrt{100}$
- (f) $\sqrt{121 - 100}$
- (g) $\sqrt{121} - \sqrt{100}$
- (h) $(\sqrt{5})^2$
- (i) $\sqrt{8} \cdot \sqrt{2}$
- (j) $\sqrt{\dfrac{169}{9}}$

(k) $-\sqrt{\dfrac{900}{289}}$

(l) $\dfrac{3}{\sqrt{6}}$

(m) $\dfrac{5}{\sqrt{5}}$

(n) $\dfrac{7}{\sqrt{17}}$

(o) $\dfrac{1}{\sqrt{11}}$

(p) $\sqrt{75}$

(q) $\sqrt{200}$

(r) $\sqrt{1000}$

(s) $\sqrt{\dfrac{324}{25}}$

(t) $\sqrt{\dfrac{1600}{49}}$

(6) Simplify each square root:

(a) $\sqrt{x^8}$

(b) $\sqrt{x^9} \quad (x \geq 0)$

(c) $\sqrt{\dfrac{18m^5n^6}{p^{12}q^7}}$

(d) $\sqrt{x^{12}y^{30}z^{22}}$

(e) $\sqrt{\dfrac{x^{22}}{y^{12}z^{24}}}$

(7) Rationalize the denominator:

(a) $\dfrac{1}{\sqrt{3}}$

(b) $\dfrac{1}{\sqrt{5}}$

(c) $\dfrac{2}{\sqrt{7}}$

(d) $\dfrac{2}{\sqrt{6}}$

(e) $\dfrac{abc}{\sqrt{abc}}$

(f) $\dfrac{1}{1+\sqrt{3}}$

(g) $\dfrac{1}{2-\sqrt{5}}$

(h) $\dfrac{1}{\sqrt[4]{27}}$

(i) $\dfrac{2}{\sqrt[5]{16}}$

(j) $\dfrac{15}{\sqrt[3]{5a^2b}}$

22. Operations on radical expressions

(1) Perform the following operations and simplify:

(a) $\sqrt{2} + 3\sqrt{2}$

(b) $\sqrt[3]{4} - 5\sqrt[3]{4}$

(c) $\sqrt[4]{9} - 7\sqrt[4]{9} + 12\sqrt[4]{9}$

(d) $\sqrt[3]{15} - \sqrt[5]{15}$

(e) $\sqrt{75} - \sqrt{3}$

(f) $2\sqrt{18} + 5\sqrt{2} - 4\sqrt{50}$

(g) $2\sqrt{27} - 7\sqrt{12}$

(h) $3\sqrt{20} + 2\sqrt{45} + 8\sqrt{80}$

(i) $\sqrt{3}\left(\sqrt{3} + \sqrt{5}\right)$

(j) $\sqrt{5}\left(3 - \sqrt{5}\right)$

(k) $\sqrt{5}\left(2\sqrt{5} - 2\sqrt{3}\right)$

(l) $(\sqrt{12} + \sqrt{5})(\sqrt{12} - \sqrt{5})$

(m) $(\sqrt{5})^2 + 2(\sqrt{5})(\sqrt{3}) + (\sqrt{3})^2$

(n) $(\sqrt{5} + \sqrt{3})^2$

(o) $(\sqrt{5} - \sqrt{3})^2$

(p) $(\sqrt{5})^2 + (\sqrt{3})^2$

(q) $(\sqrt{5})^2 - (\sqrt{3})^2$

(2) Rationalize the denominator:

(a) $\dfrac{1}{\sqrt{3}}$

(b) $\dfrac{1}{\sqrt{5}}$

(c) $\dfrac{1}{\sqrt{11}}$

(d) $\dfrac{1}{3\sqrt{2}}$

(e) $\dfrac{1}{2\sqrt{3}}$

(f) $\dfrac{4}{4-\sqrt{5}}$

(g) $\dfrac{3}{6+\sqrt{5}}$

23. Complex Numbers

$$i = \sqrt{-1}$$

Evaluate:

(1) $\sqrt{-4}$

(2) $\sqrt{-9}$

(3) $\sqrt{-16}$

(4) $\sqrt{-25}$

(5) $\sqrt{-\frac{169}{9}}$

(6) $\sqrt{-\frac{25}{49}}$

(7) $\sqrt{-5}$

(8) $\sqrt{-3}$

(9) $\sqrt{-12}$

(10) $\sqrt{-75}$

(11) i^2

(12) i^3

(13) i^4

(14) i^9

(15) i^{12}

(16) i^{30}

(17) i^{100}

(18) i^{110}

(19) $(3+2i)+(5+4i)$

(20) $(3+2i)-(5+4i)$

(21) $(3+2i)(5+4i)$

(22) $(3+2i) \div (5+4i)$

(23) $(2-7i)+(3+i)$

(24) $(2-7i)-(3+i)$

(25) $(2-7i)(3+i)$

(26) $(2-7i) \div (3+i)$

(27) $\left(\frac{2}{3}+\frac{i}{5}\right)+\left(\frac{1}{4}-\frac{1}{3}i\right)$

(28) $\left(\frac{2}{3}+\frac{i}{5}\right)-\left(\frac{1}{4}-\frac{1}{3}i\right)$

(29) $\left(\frac{2}{3}+\frac{i}{5}\right)\left(\frac{1}{4}-\frac{1}{3}i\right)$

(30) $\left(\frac{2}{3}+\frac{i}{5}\right) \div \left(\frac{1}{4}-\frac{1}{3}i\right)$

24. Completing the square and the quadratic formula

Recall:

$(x+y)^2 =$

$(x-y)^2 =$

(1) Fill in the blanks:
- (a) $x^2 + 6x = x^2 + 6x + 9 - 9 = (x+3)^2 - 9$.
- (b) $x^2 - 6x =$____.
- (c) $x^2 + 4x =$____.
- (d) $x^2 - 4x =$____.
- (e) $x^2 + 2x =$____.
- (f) $x^2 - 2x =$____.
- (g) $x^2 + 8x =$____.
- (h) $x^2 - 8x =$____.
- (i) $x^2 + 3x =$____.
- (j) $x^2 - 3x =$____.
- (k) $x^2 + x =$____.
- (l) $x^2 - x =$____.
- (m) $x^2 + 7x =$____.
- (n) $x^2 - 7x =$____.
- (o) $x^2 + kx =$____.

(2) Solve by completing the square:
- (a) $x^2 - 6x + 5 = 0$.

  Solution:
  - Subtract 5 from both sides
  - Add 3^2 to both sides. (Can you guess why 3^2?)
  - Simplify both sides.
  - Is the left hand side a square formula? If so, then write it down.
  - Take square root of both sides (remember to consider the positive and negative options)
  - Solve for x. There should be two answers.

(b) $x^2 - 8x - 15 = 0$

Solution:

- Add 15 to both sides
- Add 4^2 to both sides. (Can you guess why 4^2?)
- Simplify both sides.
- Is the left hand side a square formula? If so, then write it down.
- Take square root of both sides (remember to consider the positive and negative options)
- Solve for x. There should be two answers.

(c) $x^2 - 10x + 18 = 0$.

Solution:

- Subtract 18 from both sides
- Add $(-5)^2$ to both sides. (Can you guess why $(-5)^2$?)
- Simplify both sides.
- Is the left hand side a square formula? If so, then write it down.
- Take square root of both sides (remember to consider the positive and negative options)
- Solve for x. There should be two answers.

(d) $x^2 - 12x + 21 = 0$

(e) $x^2 - 3x - 7 = 0$

(f) $x^2 - 5x + 3 = 0$

(g) $2x^2 - 5x + 3 = 0$ (First divide by 2 throughout and then proceed just as above).

(h) $3x^2 + 11x - 4 = 0$ (What will you first divide by?)

(i) $5x^2 - 17x - 6 = 0$

(3) We now have enough practice to work on the **general** case and derive the **Quadratic formula**.

Solve for x in $ax^2 + bx + c = 0$ where $a \neq 0$. (Do not let the variables trouble you).

Solution:

- Divide by ____________ throughout.

- Subtract ____________ from both sides

- Add ____________ to both sides.

- Simplify both sides.

- Is the left hand side a square formula? If so, then write it down.

- Take square root of both sides (remember to consider the positive and negative options)

- Solve for x. There should be two answers.

(4) The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Does your formula from problem (3) look like this one? If not, then can you simplify your formula to look like this one?

(5) Solve for x by using the quadratic formula and **simplify**:

(a) $x^2 - 6x + 5 = 0$

(b) $x^2 - 8x - 15 = 0$

(c) $x^2 - 10x + 21 = 0$

(d) $x^2 - 9x + 18 = 0$

(e) $2x^2 - 5x + 3 = 0$

(f) $3x^2 + 11x - 4 = 0$

(g) $2x^2 - 9x - 1 = 0$

(h) $5x^2 - 17x - 6 = 0$

(i) $4x^2 - 12x - 3 = 0$

(j) $3x^2 - 9x + 4 = 0$

(k) $5x^2 + 2x + 2 = 0$

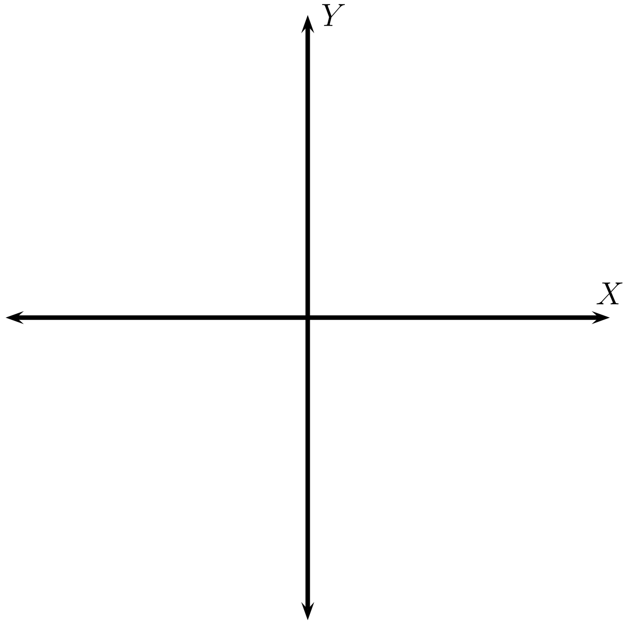
(l) $3x^2 - 12x + 7 = 0$

(m) $x^2 - \frac{3}{2}x - \frac{7}{2} = 0$

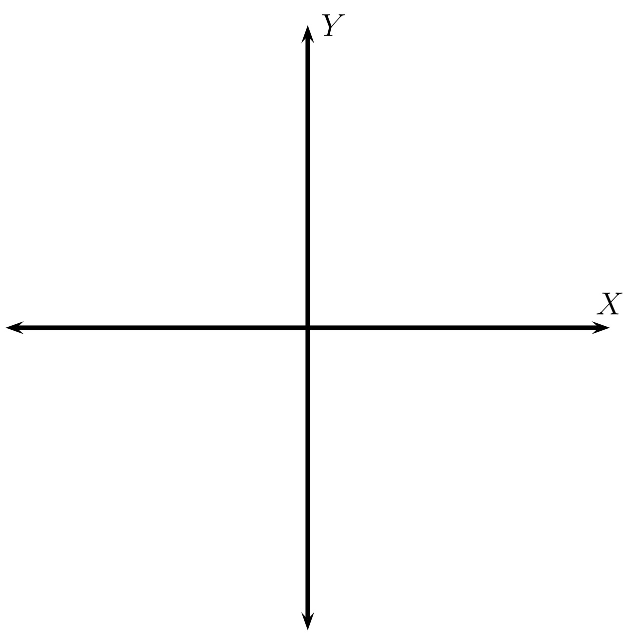
25. Introduction to Parabolas

(1) Draw the parabola given by the given equation. What is the vertex? Does the parabola open up or down? What are its X and Y intercepts? What is its axis of symmetry, and give two points on the parabola symmetric with respect to the axis of symmetry. What is the domain and range for this graph?

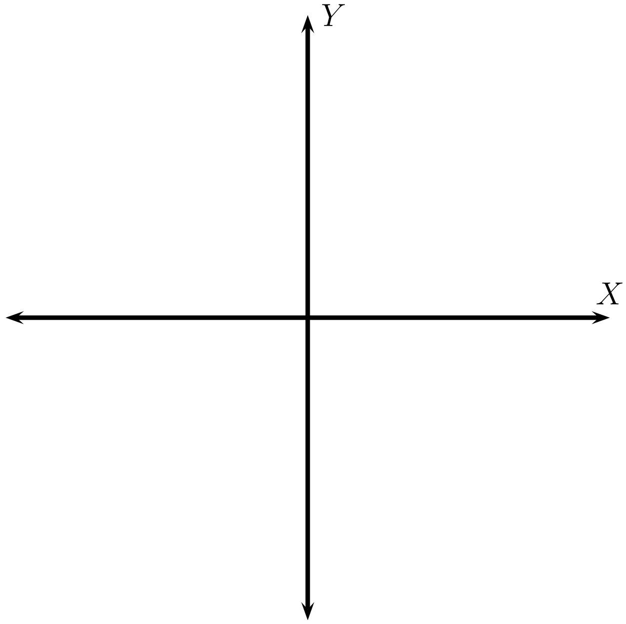
(a) $y = x^2$



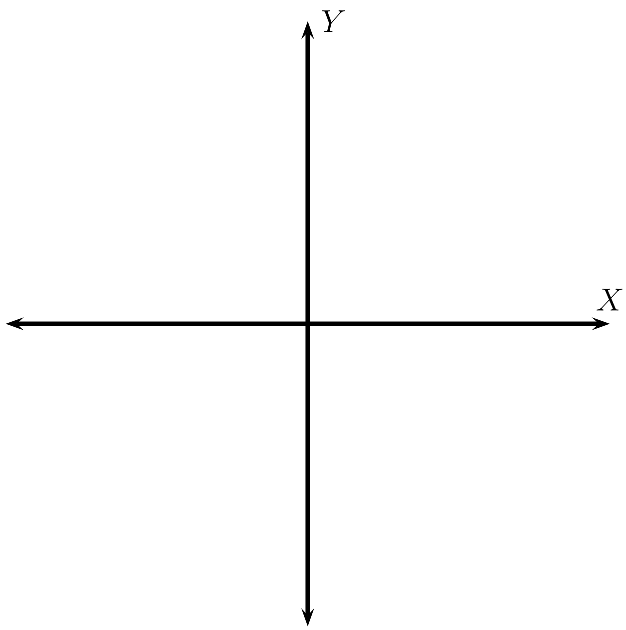
(b) $y = -x^2$



(c) $y = (x+3)^2$

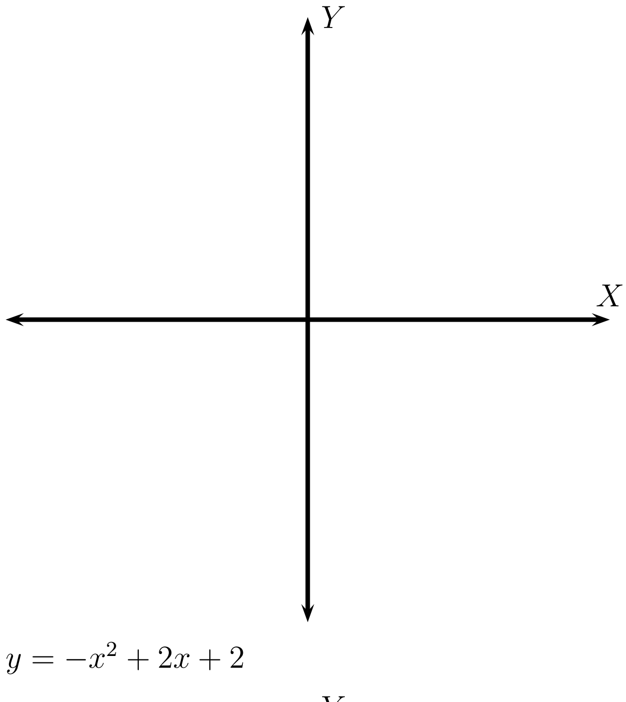


(d) $y = -(x-4)^2 + 2$.

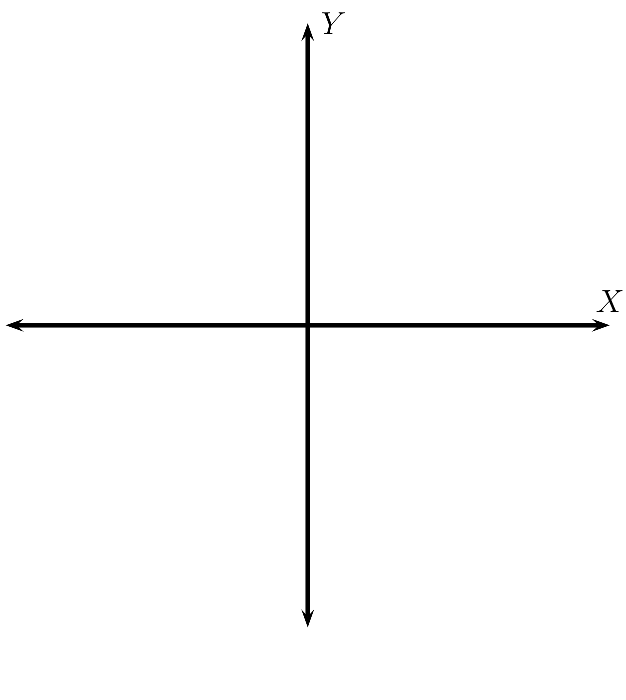


In general, the curve given by the equation $y = ax^2 + bx + c$ for $a \neq 0$ is a parabola with vertex whose x coordinate is given by $x = \dfrac{-b}{2a}$, and which opens up if $a > 0$, or opens down if $a < 0$.

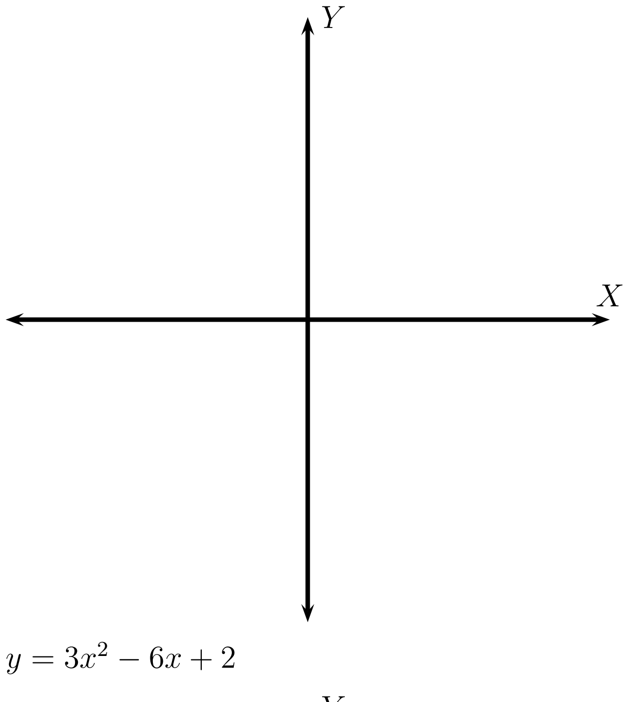
(e) $y = x^2 - 2x$



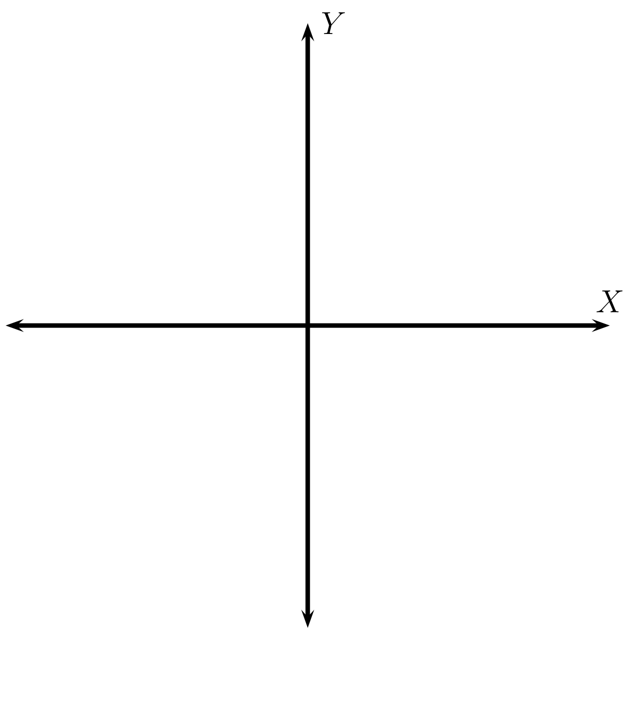
(f) $y = -x^2 + 2x + 2$



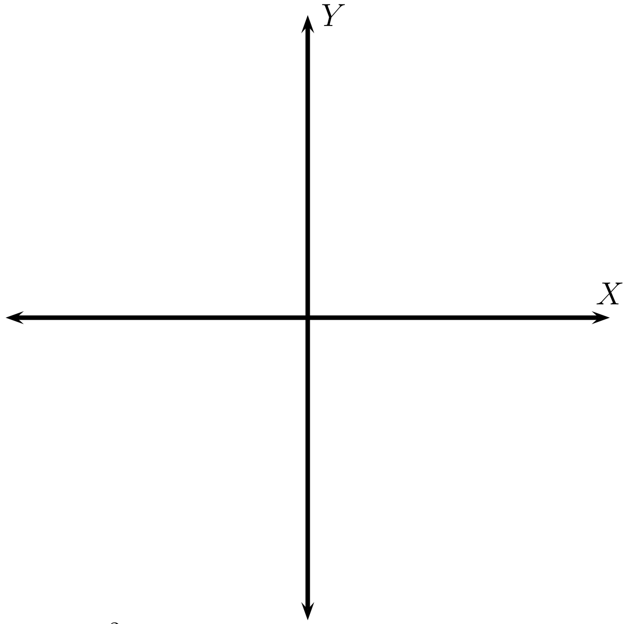
(g) $y = 3x^2 - 6x$



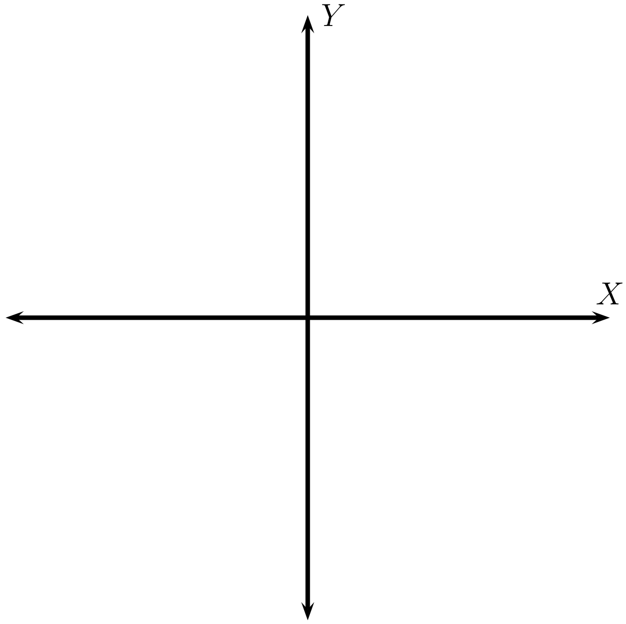
(h) $y = 3x^2 - 6x + 2$



(i) $y = -3x^2 + 12x - 1$



(j) $y = -\dfrac{x^2}{3} - 2x - 2$



(2) A projectile is launched into the air from the surface of planet A. On planet A, the height of any projectile y given in feet is determined by the equation $y = -6t^2 k v_0 t$, where t is time in seconds and v_0 is the initial velocity of the object in feet per second. If the projectile is launched from the ground level with an initial velocity of 400 feet per second, then how many seconds will it take for the projectile to reach a height of 2506 feet?

(3) At a local frog jumping contest, Rivet's jump can be approximated by the equation $y = -\frac{x^2}{4} + x$ and Croak's jump can be approximated by $y = -3x^2 + 6x$ where $x =$ length of the jump in feet and $y =$ height of the jump in feet.

- Which frog jumped higher? How high did it jump?

- Which frog jumped farther? How far did it jump?