MTH 28.5 LECTURE NOTES (Ojakian)

Topic 23: Lines

OUTLINE

References: (**Algebra Book**: pages 95-105; **Statistics Book**: None)

1. Linear versus non-linear equations
2. Lines
3. Intercepts and Slope

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1. Lines and Linear Equations

Definition 1. *An equation in two variables (x and y) is called **linear** if can be simplified to one of these three forms (M, B, and C are fixed numbers):*

- $y = Mx + B$ *(any NOT horizontal/vertical)*
- $y = B$ *(horizontal)*
- $x = C$ *(vertical)*

Theorem 1.

- *The graph of a linear equation is a line.*
- *Any line can be described with some linear equation.*

PROBLEM 1. *For each equation, guess which ones have a graph that is a line, and which ones have a graph that is **not** a line.*

(a) $y = -x$

(b) $y = x^2 + 3$

(c) $y = 2 + x$

(d) $y = -3x^5 - 10$

2. Y-Intercept

Note: We ignore the details of X-intercept ...

(a) Recall the definition of $x-$intercepts and $y-$intercepts.

(b) To find y-intercepts (for line $y = Mx + b$):
Set $x = 0$, then solve for y.

PROBLEM 2. *For each equation, find the y-intercepts of its graph.*

i. $y = 3x + 3$

ii. $y = (1/2)x - 5$

3. Graphing a line

PROBLEM 3. *Consider problem 2. Graph each of the equations.*

(a) To graph a line:
   i. Find any two points.
   ii. Connect the points by a straight line

4. Slope

Slope: A number that measures the "steepness" of a line.

(a) Finding slope by "lining up the points"
   i. Line up points and subtract
   ii. Get the $y-$change
   iii. Get the $x-$change.
   iv. Slope $= \dfrac{y - \text{change}}{x - \text{change}}$
   v. Note: Be careful of sign!

(b)

PROBLEM 4. *Suppose a line contains the following points:* $(0, 1), (2, 5), (-1, -1)$. *Find its slope.*

Theorem 2. *The slope of a line is the same, no matter what two distinct points are used to compute it.*

PROBLEM 5. *Find the slope of the line* $y = 2x + 3$

***PROBLEM* 6.** *Describe the difference between a line with positive slope versus a line with negative slope.*

5. Slope and intercept in the line equation

***PROBLEM* 7.** *Based on the above problems, guess a fast way to determine the slope of a line.*

(a) Line with slope m and y-intercept b has equation $y = mx + b$.
   i. Example: From equation of line to slope and intercept.
   ii. Example (reverse): From slope and intercept to equation of line.

(b) Using M and B to graph a line
   i. Use the y-intercept as one point.
   ii. Use the slope to find a second point:
      A. Write slope as $\dfrac{(+ \text{ or } -)\ V}{H}$
      B. Start at the y-intercept
      C. Move to the right H
      D. Move up or down V (up for positive slope, down for negative)