MTH 23.5 LECTURE NOTES (Ojakian)

Topic 22: Hypothesis Testing

OUTLINE

References (**Algebra Book**: None; **Statistics Book**: ch 9)

1. Hypothesis Testing
2. P-Values

Note: We will not refer to the "sample test statistic"
We are only covering the case where the standard deviation is known

1. Confidence Intervals versus Hypothesis Testing
   (a) Confidence Interval: Estimate the value of the mean (with some confidence)
   (b) Hypothesis Testing: Make decision about the value of the mean (with some confidence)
2. Hypothesis Testing: The Setup
   (a) First Example:
   See Example 1 from Section 9.1 (5th edition, p. 353)
      i. Null Hypothesis (H_0): The hypothesis to test
      ii. Alternative Hypothesis (H_1 or H_a)
   (b) Example:
   Section 9.1 (page 465): Example 9.2 and Try-It 9.2
   (c) Example:
   Section 9.1 (page 465): Example 9.3 and Try-It 9.3
   (d) Example:
   Exercise 7 (a, b, c, d) from secion 9.1 (5th edition, p. 364)
   (e) General points
      i. Null Hypothesis is an "Equality Statement"
      ii. Alternative Hypothesis can choosen to be any of following:
         A. Not equal ("two-tailed")
         B. Less than ("left-tailed")
         C. Greater than ("right-tailed")

3. P-Values

Probability the sample mean is as extreme as it is under the assumption of the null hypothesis.

Example: See example 2 (Section 9.1 of 5th edition, p.355)

(a) Starting point: We have a normal distribution x with an **unknown** mean μ and a **known** standard deviation σ (or may be approximately normal).

(b) The Assumption: Assume that the null hypothesis ($H_0 : \mu = 115$) is true for all calculations. What are the implications of this assumption???

(c) Consider the sampling distribution $\overline{x}$.

i. Under our assumption, its mean is 115

ii. Under our assumption calculate the standard deviation as usual: $\sigma/\sqrt{n}$

(d) $H_1 : \mu < 115$: P-Value = Probability of being less than $\overline{x}$ = norm.dist($\overline{x}$, μ, $\sigma/\sqrt{n}$, true)

(e) $H_1 : \mu > 115$: P-Value = Probability of being more than $\overline{x}$ = ... Do "1 minus ..."

(f) $H_1 : \mu \neq 115$: P-Value = Probability of being less than $\overline{x}$ or more than $\overline{x}$ = "TWO times left-tail case OR right-tail case" (which one, depends on the question!)

(g)

PROBLEM 1. *From section 9.1 (5th Edition): Do exercise 5.*
Then compute the 3 kinds of P-Values for the assumption of a standard deviation of 5kg, and various samples.

4. Hypothesis Testing - Full Problem

(a) Choose H_0 and H_1 (the null hypothesis and the alternative hypothesis)

(b) Choose α (the *Level of Significance*).
Typical values: 0.05 and 0.01.

(c) Calculate the P-Value: p

(d) If $p > \alpha$ then accept H_0. If $p < \alpha$ then reject H_0 (and accept H_1)

(e) Error types:

i. Type 1: Rejecting H_0 when it is true.

ii. Type 2: Rejecting H_1 when it is true.

(f)

PROBLEM 2. *From section 9.1 (5th Edition): Do exercise 10.*

PROBLEM 3. *From section 9.1 (5th Edition): Do exercise 11.*

PROBLEM 4. *From section 9.1 (5th Edition): Do exercise 13.*