MTH 23.5 LECTURE NOTES (Ojakian)

Topic 15: Probability and Counting

OUTLINE

References (**Algebra Book**: None; **Statistics Book**: 4.3)

1. Sets/Combinations
2. Sequences

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1. Sets

   (a) $\binom{n}{r} = C_{n,r} = nCr =$ number of subsets of S of size r (where S is any set with n elements).

   (b) Examples using above definition.

      i. Find $C(6,1)$.

      ii. Find $C(4,2)$ by listing all the appropriate subsets.

   (c) Formula: $\binom{n}{r} = \dfrac{n!}{r! \cdot (n-r)!}$ (note: $k!$ is "k factorial")

   Apply to the above.

   (d)

   PROBLEM 1. *Calculate the following:*

   i. $3!$

   ii. $1!$

   iii. $0!$

   iv. $5 \cdot (10-8)!$

   v. $C_{3,2}$

   vi. $4!$

   vii. $\binom{4}{1}$

2. Sequences

   Key principle: To find the number of sequences (ordered!) of some type, for each spot find the number of options and multiply.

   Symbolicaly stated: The number of sequences where position i has n_i options is $n_1 \cdot n_2 \cdots$

   (a) How many 4-letter words can be formed just using vowels? (there are 5 vowels)

   (b) How many 4-letter words can be formed just using vowels if no repetitions are allowed?

3. With Probability

   (a) What is the probability your initial blackjack hand contains only face cards?

   (b) What is the probability of a flush in poker?

   (c) Suppose you roll a die 4 times in a row. What is the probability that all the rolls are even?