MTH 23.5 LECTURE NOTES (Ojakian)

Topic 12: Probability- Operations on Events

OUTLINE

References (**Algebra Book**: None; **Statistics Book**: 3.2, 3.3, 3.5 (Venn diagrams))

1. And, Or, Complement operations
2. Mutually exclusive
3. Independant

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1. Operations on Events

   (a) "*And*" Operation (Intersection of events)

   PROBLEM 1. *Consider a 6-sided die which you roll once.*

   i. *Let A be the event: The roll is even.*
   Let B be the event: The roll is less than 5.
   Find $P(A$ and $B)$.

   ii. *Find P(Even and Odd)*

   (b) "*Or*" Operation (Union of events)

   PROBLEM 2. *Do the last problem, but replace the word "and" by the word "or".*

   (c) Complement (two notations: "c" or over bar)

   PROBLEM 3. *Section 5.1 from MY Stats Book 5th Edition (p. 165): Exercise 10*

2. Formula for complement

   $P(E) = 1 - P(E^c)$

3. Formulas for OR

   Consider the following attempt at a rule:

   $$P(A \text{ or } B) = P(A) + P(B)?!?!$$

   Is the rule true or false? When?? Use some examples to get evidence of your view.

   (a) Define: Mutually Exclusive (or disjoint). Then above rule true.

   (b) Rule when A and B are NOT mutually exclusive?
   Consider: Basic question.

   PROBLEM 4. *You roll two fair 4-sided dice, one green and one blue.*

   i. *What is the probability of getting a sum of 3?*

   ii. *What is the probability of getting a sum of 6?*

   iii. *What is the probability of getting a sum of 3 or 6?*

iv. What is the probability of getting a sum of 6 or having at least one of the die being a 4.

4. Formulas for AND

Independence: Two events are independent if the occurrence or nonoccurrence of one event does not change the probability that the other event will occur.

(a) Intuitive Examples: Independent or not?

It rains in Manhattan versus It rains in The Bronx.

It rains in Manhattan versus It rains in San Juan.

(b) Mathematical Examples. Independent or not?

Roll a blue die and a red die. The blue die being 5 versus The red die being 2.

Roll one die: Rolling a two versus Rolling an even number.

Roll one die: Rolling an odd number versus Rolling an even number.

(c) Mathematical Definition.

Two events A and B are independent if $P(A \text{ and } B) = P(A) \cdot P(B)$

PROBLEM 5. *Roll one die one time. Let A be: The value is 2. Let B be: The value is an even number. Are A and B independent or not?*

PROBLEM 6. Use the fact that the roll of one die is independent of the roll of another die. *Suppose two dice are rolled, one blue and one red; what is the probability that the blue die is a 3 and the red die is a 5?*

PROBLEM 7. *Suppose two dice are rolled; what is the probability that one die is a 3 and the other die is a 5?*

PROBLEM 8. *Suppose a die is rolled three times; what is the probability that the first roll is a 3, the second roll is a 1, and the final roll is a 1?*

5. Combining operations

PROBLEM 9. *Section 5.2 from MY Stats Book 5th Edition (p. 181): Exercise 9 (Dice Problem)*