MTH 23.5 LECTURE NOTES (Ojakian)

Topic 11: Solving Equations

OUTLINE
References: Algebra Book

1. Theoretical Background
   (a) What it means to solve an equation
2. Solving Equations
   (a) By Guess-and-Check
   (b) Using Algebraic Tools
3. Solving for variable in formula

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1. Solution Sets and Solving

Definition 1. *Given an equation, a* ***solution*** *is a number that make it true (i.e. plug it in and both sides evaluate to the same number).*

PROBLEM 1. *For each equation, find all its solutions.*

(a) $x + 5 = 15$
(b) $x + 2x = 3x$
(c) $y + y = y$
(d) $x = x + 1$
(e) $x^2 = 25$

Theorem 1. *Given an equation, there are three possibilities:*

(a) It is always true (i.e. it is an identity), or
(b) It is sometimes true, or
(c) It is never true (has "No Solution")

Definition 2. *To* ***solve*** *an equation means to find ALL the solutions of the equation.*

2. Solving Equations by Guess-and-Check

(a) Checking a possibility.

PROBLEM 2. *Consider the equation* $x^2 = x - x^2$.

i. Is $x = 0$ *a solution?*
ii. Is $x = -1/3$ *a solution?*
iii. Is $x = 1/2$ *a solution?*

(b) Guessing and Checking

PROBLEM 3. *Consider the equation* $x + 3 = -5$. *Solve it by guessing and checking.*

3. Tools for solving equations

(a) Combining like terms

(b) Add the same quantity to both sides.

(c) Subtract the same quantity from both sides.

(d) Mutliply both sides by the same quantity.

(e) Divide both sides by the same quantity (EXCEPT! ... If the quantity = ???)

4. Solving for a variable

Given an equation with many variables, solve for the requested one.

(a) Do the same thing as when you solve any equation, with the goal of isolating the requested variable.

(b) Examples of well known formula (solve for some variable)

i. $A = L \cdot W$ (Area of Rectangle)

ii. $A = \dfrac{b \cdot h}{2}$ (Area of Triangle)

iii. $P = 2L + 2W$ (Perimeter of Rectangle)

iv. $F = C \cdot \dfrac{9}{5} + 32$ (Fahrenheit vs. Celsius)

v. $C = 2\pi r$ (Circumference of a circle)