MTH 23.5 LECTURE NOTES (Ojakian)

Topic 9: Arithmetic with Fractions

OUTLINE

References (**Algebra Book**: pages 14-17; **Statistics Book**: None)

1. Multiplying and Dividing Fractions/Decimals
2. Adding and Subtracting Fractions/Decimals
3. Least Common Multiple (LCM)

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1. Conversion: Mixed Fractions

(a) Fraction $\to$ Mixed

Divide - Use quotient as integer part and use remainder as the top

PROBLEM 1. *Convert* $13/3$ *to a mixed number (use both methods).*

(b) Mixed $\to$ Fraction

i. Find the product of the integer part and the bottom

ii. Add to the top

PROBLEM 2. *Write* $3\frac{2}{7}$ *as an improper fraction.*
What happens if it were negative?

2. Multiplication of Fractions

(a) Multiply tops

(b) Multiply bottoms

(c) Simplify

PROBLEM 3.

i. Compute the following: $\frac{1}{2} \cdot \frac{1}{5}$

ii. Convert the fractions to decimal and then try the calculation.

iii. Does the answer make sense? (it should!)

PROBLEM 4. *Compute the following:*

i. $\frac{2}{3} \cdot \left(-\frac{1}{5}\right)$

ii. $\frac{-2}{-3} \cdot \left(\frac{-1}{5}\right)$

iii. $\frac{2}{5} \cdot \frac{15}{8}$

iv. $\frac{49}{88} \cdot \frac{-11}{7}$

3. Dividing Fractions

(a) Find reciprocal of second fraction

(b) Then multiply

PROBLEM 5.

i. Compute the following: $\frac{3}{2} \cdot \frac{1}{4}$

ii. Convert the fractions to decimal and then try the calculation.

iii. Does the answer make sense? (it should!)

PROBLEM 6. *Compute the following:*

i. $-\frac{50}{12} \div \frac{-5}{3}$ *(Simplify before calculating!)*

ii. $\frac{1}{5} \div \frac{10}{7}$ *(Be careful on simplifing!)*

4. Adding and Subtracting Fractions

(a) When denominators are the same.

Recall interpretation: A/B means to take A steps of size $1/B$ each.

PROBLEM 7. *By* **counting on the number line** *compute the following:*

i. $\frac{3}{4} + \frac{2}{4}$

ii. $-\frac{3}{4} + \frac{-1}{4} + \frac{5}{4}$

iii. Convert the fractions to decimals and try it. Make sense? (it should!)

5. Denominators NOT the same when Adding and Subtracting Fractions

(a) Method.

i. Make the denominators the same! (i.e. find a **common denominator**)

ii. Then add or subtract tops

iii. Leave the bottom the same

PROBLEM 8. *Compute the following:*

A. $\frac{7}{20} + \frac{2}{5}$

B. $\frac{1}{-2} - \frac{-2}{3} + -\frac{2}{5}$

6. Quick way to get a common denominator

(a) Just multiple all the bottoms together to get the new bottom! (may get large numbers!)

(b) Thus: Multiply each fraction's top and bottom by the *other* bottoms

7. To find LEAST common denominator

(a) Least Common Multiple

Definition. *The **Least Common Multiple** (**L.C.M.**) of a group of integers is the smallest integer that all the integers in the group divide into evenly.*

Way to find LCM: Count by multiples till the first common multiple is reached.

PROBLEM 9. *Find the LCM for each group of integers.*

i. 3, 10

ii. 4, 6

iii. 120, 10, 5

(b) Addition/Subtraction with large denominators

Definition. *Given a group of fractions, the **Least Common Denominator** (**L.C.D.**) is the LCM of the denominators.*

The LCD is the smallest common denominator that can be used for the addition and subtraction of fractions.

PROBLEM 10.

i. $\frac{5}{6} + \frac{1}{4}$

ii. $\frac{2}{7} + \frac{3}{14}$

8. Fractions and Order Of Operations

PROBLEM 11.

(a) $\frac{1}{3} + \frac{1}{2} \cdot \frac{2}{3}$

(b) $\frac{2}{7} \cdot \frac{14}{6} + \frac{2}{4} \div \frac{3}{8}$

9. Fractions and Algebra

PROBLEM 12.

(a) $4x \quad$ *if* $x = \dfrac{1}{2}$

(b) $4x \quad$ *if* $x = 0.5$

(c) $4x \quad$ *if* $x = -\dfrac{1}{2}$

(d) $2a - b \quad$ *if* $a = \dfrac{1}{4}$, $b = \dfrac{3}{2}$

(e) $2a - b \quad$ *if* $a = -1$, $b = -\dfrac{3}{2}$

(f) $x^2 \quad$ *if* $x = \dfrac{2}{5}$

(g) $x^2 \quad$ *if* $x = -\dfrac{2}{5}$

(h) $x^2 + y^2 \quad$ *if* $x = \dfrac{3}{2}$, $y = 1$

(i) $x^2 - y^2 + \dfrac{7}{10} \quad$ *if* $x = -\dfrac{3}{4}$, $y = \dfrac{1}{2}$

10. Applications

PROBLEM 13. *Find the area and perimeter of a rectangle of width 7 amd height 3/14.*

PROBLEM 14. *If I pay every student in class 5.25 to study, how much do I lose? And if I only have 30 dollars, how many students can I pay?*