

SOLUTION
BRONX COMMUNITY COLLEGE
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DEPARTMENT OF MATHEMATICS AND
COMPUTER SCIENCE

CSI 35

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YOUR NAME (first, then last):

Exam 1

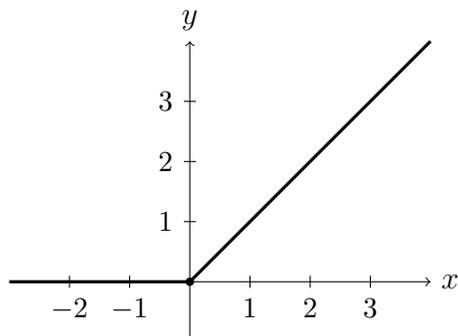
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Directions: Write your responses in the provided space. To get full credit you **must** show all your work. Simplify your answers whenever possible. Be certain to indicate your final answer clearly. **No** electronic devices are allowed (i.e. no calculators, no phones, no smart watches, etc) - using one during the exam will result in at least a failure on this test. Each question is worth 10 points (total scaled to 100).

1. Define a Python function which takes one numerical input. If the input is negative, it returns 0; otherwise it just returns back the same input. Graph this function.

Note: This function is famous in the AI neural net work; called the “ReLU” function.
SOLUTION.

```
def my_function(x):  
    if x < 0:  
        return 0  
    else:  
        return x
```



2. Write a while-loop that sums the numbers from 10 to 200.
SOLUTION.

```
                                # Initialize variables

total = 0
num = 10

                                # While-loop to sum numbers from 10 to 200

while num <= 200:
    total += num
    num += 1

                                # The final result is stored in total

print(total)
```

3. Let h be defined by recursion as follows: $h(3) = 2$ and $h(n + 1) = 2h(n) - 3$. Write the definition of a Python function that takes one integer as input and returns the output of h on that input.
SOLUTION.

```
def h(n):
    if n == 3:
        return 2
    else:
        return 2 * h(n - 1) - 3
```

4. Recursively define f as follows: $f(1) = 0$, $f(2) = 2$ and $f(n) = f(n - 1) + 2 \cdot f(n - 2)$.

(a) Evaluate $f(3)$ and $f(5)$.

(b) Write a recursive Python function that computes f (you may assume the function input is a positive integer).

SOLUTION:

(a) $f(3) = f(2) + 2 \cdot f(1) = 2 + 2 \cdot 0 = 2$

$f(5) = f(4) + 2 \cdot f(3) = (f(3) + 2 \cdot f(2)) + f(3) = (2 + 2 \cdot 2) + 2 = 6 + 2 = 8$

(b) def $f(n)$:

 if $n == 1$:

 return 0

 elif $n == 2$:

 return 2

 else:

 return $f(n - 1) + 2 * f(n - 2)$

5. Consider the proposition: 5 divides $n^5 - n$ for $n \geq 2$.

In order to prove this by induction you should 1) state what statement $P(n)$ you are doing induction on, 2) show $P(n)$ is true for the base case, and 3) do the inductive step.

Just do (1) and (2). That is, state exactly what $P(n)$ is, then state and prove the base case. Do **not** do the inductive step.

SOLUTION.

1) The statement $P(n)$ we will use for induction is: $5 \mid n^5 - n$.

2) Base Case: We will show that $P(2)$ is true. For $n = 2$, we have:

$$n^5 - n = 2^5 - 2 = 32 - 2 = 30.$$

Since 30 is divisible by 5, we have $5 \mid 30$. Therefore, $P(2)$ is true.

6. Use induction to prove that $2^0 + 2^1 + 2^2 + \dots + 2^n = 2^{n+1} - 1$

SOLUTION:

Let $P(k)$ be: “ $2^0 + 2^1 + 2^2 + \dots + 2^k = 2^{k+1} - 1$ ”

Base case: $P(0)$: “ $2^0 = 2^1 - 1$ ” true since $2^0 = 1$ and $2^1 - 1 = 2 - 1 = 1$

Induction step. Given $P(k)$, we show $P(k+1)$. $P(k+1)$ states that “ $2^0 + 2^1 + 2^2 + \dots + 2^k + 2^{k+1} = 2^{k+2} - 1$ ”

By I.H. $P(k)$ we can rewrite the left side of the last as: $2^{k+1} - 1 + 2^{k+1} = 2 \cdot 2^{k+1} - 1 = 2^{k+2} - 1$, which is the right side.

7. Prove that for all positive integers n , n^2 is odd if and only if n is odd.

SOLUTION.

To prove the statement, we need to show two implications:

1. If n is odd, then n^2 is odd. 2. If n^2 is odd, then n is odd.

Proof of 1:

Assume n is odd. By definition, $n = 2k + 1$ for some integer k . Then:

$$n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$$

Since $2k^2 + 2k$ is an integer, we have $n^2 = 2m + 1$ for $m = 2k^2 + 2k$, thus n^2 is odd.

Proof of 2:

Assume n^2 is odd. Suppose for contradiction that n is even. Then $n = 2k$ for some integer k . Thus:

$$n^2 = (2k)^2 = 4k^2 = 2(2k^2)$$

This implies n^2 is even, contradicting the assumption that n^2 is odd. Hence, n must be odd.

Conclusion:

We have shown that n^2 is odd if and only if n is odd.

8. A **perfect square** is an integer that is equal to the square of another integer; for example 9 is a perfect square because $9 = 3^2$, but 18 is *not* a perfect square.

(a) Prove that if m and n are both perfect squares, then nm is also a perfect square.

(b) Prove or disprove: The sum of two perfect squares is a perfect square.

SOLUTION.

We address each part separately.

a. Prove that if m and n are both perfect squares, then nm is also a perfect square.

Let $m = a^2$ and $n = b^2$ for some integers a and b . Then:

$$nm = (a^2)(b^2) = (ab)^2$$

Since $(ab)^2$ is the square of the integer ab , nm is a perfect square.

b. Prove or disprove: The sum of two perfect squares is a perfect square.

Consider the perfect squares $1^2 = 1$ and $2^2 = 4$ (note: many other choices are valid):

$$1 + 4 = 5$$

5 is not a perfect square, as there is no integer c such that $c^2 = 5$. Therefore, the statement is disproven: the sum of two perfect squares is not necessarily a perfect square.
